Standard Order

- Given n variables, we use an n-bit expansion of the index, i, to indicate normal (true) or complement states for the variables.
- Minterms: "1" $\Rightarrow$ true; "0" $\Rightarrow$ complemented.
  - m_0 (minterm 0): $\bar{x} \cdot \bar{y} \cdot \bar{z}$; m_3 (minterm 3): $\bar{x} \cdot y \cdot z$.
- Maxterms: "0" $\Rightarrow$ true; "1" $\Rightarrow$ complemented.
  - M_0 (maxterm 0): $x + y + z$; M_3 (maxterm 3): $x + \bar{y} + \bar{z}$.
- m_i is the **complement** of M_i ($m_i = \bar{M}_i$).

Minterm and Maxterm

- **Minterms:** Terms with all variables present, combined with AND.
  1. For n variables combined with AND, there are 2^n combinations. Each unique combination is called a **minterm**.
  2. Exp: $X \cdot Y \cdot Z$, $\bar{a}bc\bar{d}$.
- **Maxterms:** Terms with all variables present, combined with OR.
  1. For n variables combined with OR, there are 2^n combinations. Each unique combination is called a **maxterm**.
  2. Exp: $X + Y + Z$, $\bar{a} + b + c + \bar{d}$.
- Exp: Two-variable terms

Index	Minterm	Maxterm
0	$\bar{x}\bar{y}$	$x + y$
1	$\bar{x}y$	$x + \bar{y}$
2	$x\bar{y}$	$\bar{x} + y$
3	xy	$\bar{x} + \bar{y}$

Sum of Minterms

- Any Boolean function can be expressed as a sum of minterms.
- Exp: $F = A + \bar{B}C$.
  - Expand the terms with missing variables and collect terms:

$$\begin{aligned} F &= A + \bar{B}C \\ &= A(B+\bar{B})(C+\bar{C}) + (A+\bar{A})\bar{B}C \\ &= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C \end{aligned}$$

  - Sum of minterms: $F = m_7+m_6+m_5+m_4+m_1 \Longrightarrow F(A,B,C) = \sum_m(1,4,5,6,7)$.
- The complement of a function contains those minterms not included in the original function.
  - $F(A,B,C) = \sum_m(1,4,5,6,7) \Longrightarrow \bar{F}(A,B,C) = \sum_m(0,2,3)$.

Minterms and Maxterms

- Minterms for three variables.

X	Y	Z	Product Term	Symbol	m0	m1	m2	m3	m4	m5	m6	m7
0	0	0	$\overline{X}\overline{Y}\overline{Z}$	m0	1	0	0	0	0	0	0	0
0	0	1	$\overline{X}\overline{Y}Z$	m1	0	1	0	0	0	0	0	0
0	1	0	$\overline{X}Y\overline{Z}$	m2	0	0	1	0	0	0	0	0
0	1	1	$\overline{X}YZ$	m3	0	0	0	1	0	0	0	0
1	0	0	$X\overline{Y}\overline{Z}$	m4	0	0	0	0	1	0	0	0
1	0	1	$X\overline{Y}Z$	m5	0	0	0	0	0	1	0	0
1	1	0	$XY\overline{Z}$	m6	0	0	0	0	0	0	1	0
1	1	1	XYZ	m7	0	0	0	0	0	0	0	1

- Maxterms for three variables.

X	Y	Z	Sum Term	Symbol	M0	M1	M2	M3	M4	M5	M6	M7
0	0	0	$X+Y+Z$	M0	0	1	1	1	1	1	1	1
0	0	1	$X+Y+\overline{Z}$	M1	1	0	1	1	1	1	1	1
0	1	0	$X+\overline{Y}+Z$	M2	1	1	0	1	1	1	1	1
0	1	1	$X+\overline{Y}+\overline{Z}$	M3	1	1	1	0	1	1	1	1
1	0	0	$\overline{X}+Y+Z$	M4	1	1	1	1	0	1	1	1
1	0	1	$\overline{X}+Y+\overline{Z}$	M5	1	1	1	1	1	0	1	1
1	1	0	$\overline{X}+\overline{Y}+Z$	M6	1	1	1	1	1	1	0	1
1	1	1	$\overline{X}+\overline{Y}+\overline{Z}$	M7	1	1	1	1	1	1	1	0

Standard Forms

- **Canonical forms** (Sum-of-Minterms or Product-of-Maxterms) have **one and only one** representation.
  - Sum-of-Minterms: $xyz + \bar{x}y\bar{z} + x\bar{y}z$, $A\bar{B} + \bar{A}B$.
  - Product-of-Maxterms: $(x + y + z)(\bar{x} + y + \bar{z})(x + \bar{y} + z)$, $(A + \bar{B})(\bar{A} + B)$
- **Standard Sum-of-Products (SOP) form:** Equations are written as **AND** terms summed with **OR** operators.
  - SOPs: $xyz + \bar{x}y\bar{z} + \bar{y}$, $A\bar{B} + \bar{A}B$
- **Standard Product-of-Sums (POS) form:** Equations are written as **OR** terms, all **AND**ed together.
  - POSs: $(x + y + z)(\bar{x} + y + \bar{z})(\bar{y})$, $(A + \bar{B})(\bar{A} + B)$
- Mixed forms: **not** SOP or POS
  - $(x\bar{y} + z)(\bar{x} + y)$, $AB\bar{C} + C(\bar{A} + B)$

Product of Maxterms

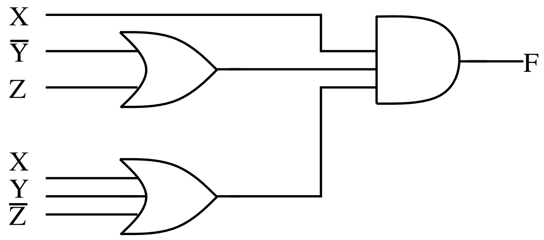
- Any Boolean function can be expressed as a product of maxterms.
- Exp: $G = AB + \bar{A}\bar{B}$.

$$\begin{aligned} AB + \bar{A}\bar{B} &= (AB + \bar{A})(AB + \bar{B}) \\ &= (\bar{A} + AB)(\bar{B} + AB) \\ &= (\bar{A} + A)(\bar{A} + B)(\bar{B} + A)(\bar{B} + B) \\ &= 1 \cdot (\bar{A} + B)(\bar{B} + A) \cdot 1 \\ &= (\bar{A} + B)(\bar{B} + A) \\ &= M_2 \cdot M_1 \end{aligned}$$

- Product of maxterms: $G = M_2 \cdot M_1 \Longrightarrow G(A, B) = \Pi_M(1, 2)$.
- The complement of a function contains those maxterms not included in the original function.
  - $G(A, B) = \Pi_M(1, 2) \Longrightarrow \bar{G}(A, B) = \Pi_M(0, 3)$.
- $G(A, B) = \Pi_M(1, 2) = \sum_m(0, 3)$; $\bar{G}(A, B) = \Pi_M(0, 3) = \sum_m(1, 2)$.

Canonical Product of Maxterms

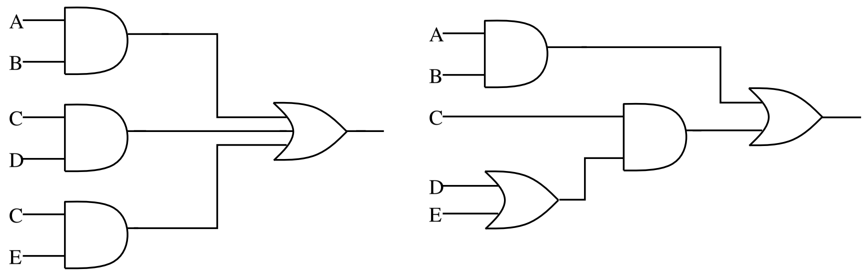
- Two-level product-of-sums implementation.
  - Exp: $F = X(\bar{Y} + Z)(X + Y + \bar{Z})$



- A canonical product-of-maxterms form implies a 2-level network of gates implementation (**1st level: OR; 2nd level: AND**).
  - Usually **not** a minimum literal Boolean expression $\Longrightarrow$ more expensive implementation.
  - Exp: $F = (A+B+C)(A+\bar{B}+C)(A+\bar{B}+\bar{C}) = (A+C)(A+\bar{B})$ (Why? Duplicate $(A+\bar{B}+C)$!)
  - The canonical product-of-maxterms form had **9 literals** and **3 terms**; the reduced POS form had **4 literals** and **2 terms**.

Canonical Sum of Minterms

- Two-level vs. three-level implementation.
  - Exp: $F = AB+CD+CE$ (2-level); $F = AB+C(D+E)$ (3-level)



(a) AB+CD+CE (b) AB+C(D+E)

- A canonical sum-of-minterms form implies a 2-level network of gates implementation (**1st level: AND; 2nd level: OR**).
  - Usually **not** a minimum literal Boolean expression $\Longrightarrow$ more expensive implementation.
  - Exp: $F = ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}\bar{B}C = A + \bar{B}C$ (Why? Duplicate $A\bar{B}C$!)
  - The canonical sum-of-minterms form had **15 literals** and **5 terms**; the reduced SOP form had **3 literals** and **2 terms**.