National Chiao Tung University
Dept. of Computer & Information Science
Intro. to Digital Systems, Spring 1998

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Handout #13
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Sample Solutions to Quiz #2

Instructions. This is a 50-minute quiz. There are five questions to be graded out of 90. Good luck!

1. Question 1. (16 pts total).

Problem	Sign–Magnitude	1's Complement	2's Complement
0 + 0 0	00000000 + 00000000 00000000	00000000 + 00000000 00000000	00000000 + 00000000 00000000
−120 + 45 ??	11111000 + 00101101 11001011		
120 + 45 ??			**overflow**
120 + −45 ??		01111000 + 11010010 01001011	01111000 + 11010011 01001011

2. Question 2. (14 pts total). We assume $Z = BC$.

D	A	Y	P_3	P_2	P_1	P_0
0	x	x	0	0	0	0
1	0	0	0	0	0	1
1	0	1	0	0	1	0
1	1	0	0	1	0	0
1	1	1	1	0	0	0

$$
\begin{aligned}
F(A,B,C,D) &= P_0 + P_3 \\
&= D\bar{A}\bar{Z} + ADZ \\
&= D\bar{A}\overline{BC} + ABCD \\
&= D\bar{A}(\bar{B}+\bar{C}) + ABCD \\
&= \bar{A}\bar{B}D + \bar{A}\bar{C}D + ABCD
\end{aligned}
$$

$$
\begin{aligned}
&= \bar{A}\bar{B}(C+\bar{C})D + \bar{A}(B+\bar{B})\bar{C}D + ABCD \\
&= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + ABCD \\
&= \Sigma_m(1,3,5,15)
\end{aligned}
$$

3. Question 3. (20 pts total).

   (a) The truth table for marjority function.

W	X	Y	Z	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

   (b) So, we get $F = wxz + wxy + wyz + wyz$ from the truth table.

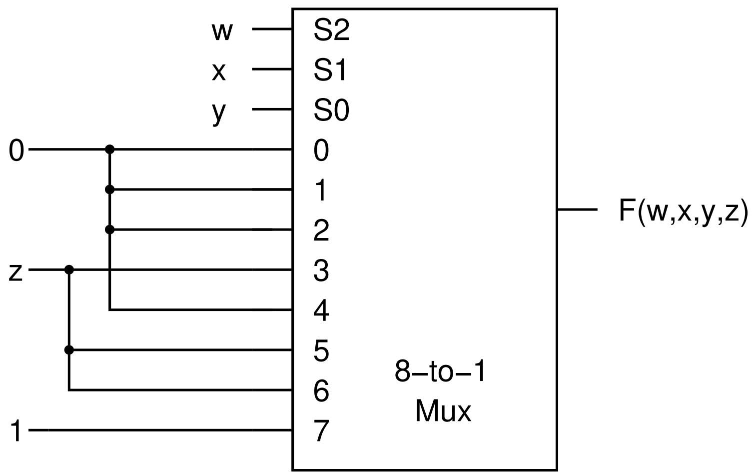


Figure 1: $F = wxz + wxy + wyz + wyz$

4. Question 4. (20 pts total).

   (a) The truth table as following.

E	X_2	X_1	X_0	Y_2	Y_1	Y_0
0	0	0	0	0	0	0
0	0	0	1	0	0	1
0	0	1	0	0	1	0
0	0	1	1	0	1	1
0	1	0	0	1	0	0
0	1	0	1	1	0	1
0	1	1	0	1	1	0
0	1	1	1	1	1	1
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	1	0	0
1	0	1	1	1	0	1
1	1	0	0	1	1	0
1	1	0	1	1	1	1
1	1	1	0	0	0	0
1	1	1	1	0	0	1

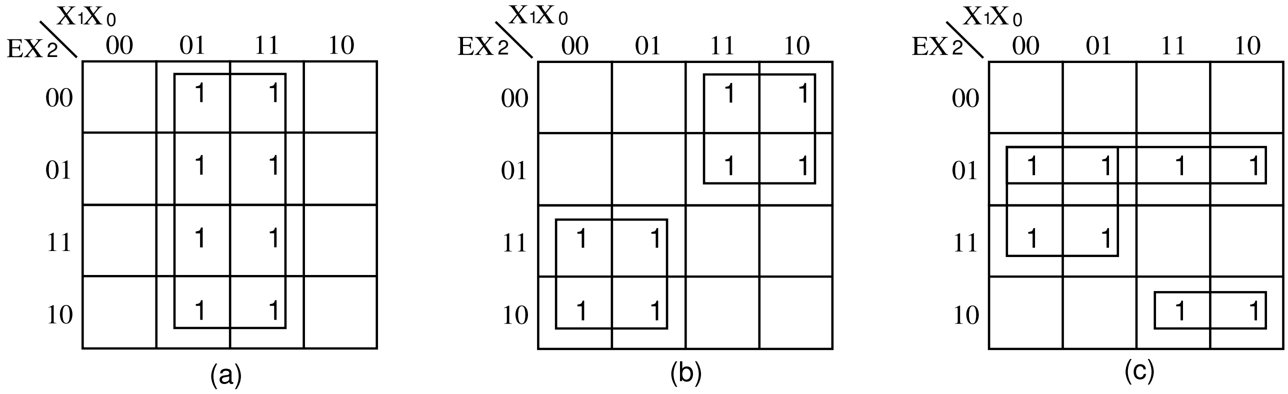
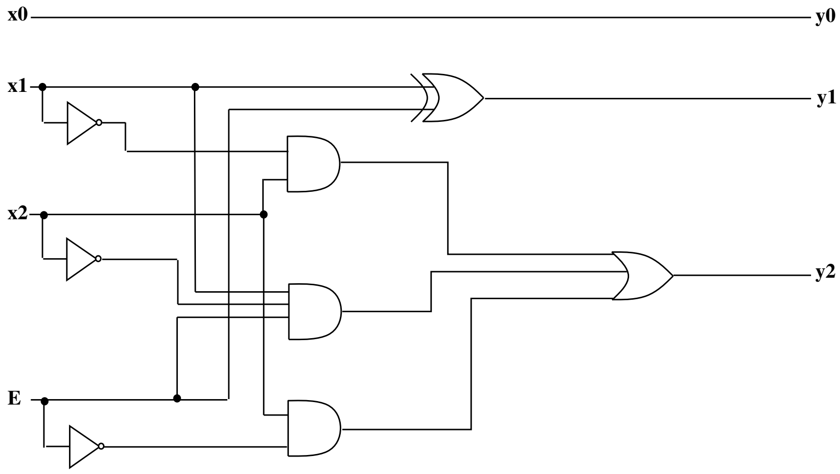


Figure 2: (a) $Y_0=X_0$; (b) $Y_1=\bar{E}X_1+E\bar{X}_1=X_1\oplus E$; (c) $Y_2=\bar{E}X_2+X_2\bar{X}_1+E\bar{X}_2X_1$.

(b) The circuit with the minumal number of gates.



5. Question 5. (20 pts total).

(a)

$$
\begin{aligned}
c_1 &= g_0 + c_0 p_0 \\
c_2 &= g_1 + c_1 p_1 \\
&= g_1 + (g_0 + c_0 p_0) \\
&= g_1 + p_1 G_0 + C_0 p_0 p_1 \\
c_3 &= g_2 + c_2 p_2 \\
&= g_2 + (g_1 + p_1 G_0 + C_0 p_0 p_1) p_2 \\
&= g_2 + g_1 p_2 + g_0 p_1 p_2 + c_0 p_0 p_1 p_2 \\
c_4 &= g_3 + c_3 p_3 \\
&= g_3 + (g_1 p_2 + g_0 p_1 p_2 + c_0 p_0 p_1 p_2) p_3 \\
&= g_3 + g_2 p_3 + g_1 p_2 p_3 + g_0 p_1 p_2 p_3 + c_0 p_0 p_1 p_2 p_3
\end{aligned}
$$

(b) we know that $A = 1010$ and $c_0 = 0$. If the propagate function $p_2 = 1$ and generate function $g_1 = 1$, we have

$$
\begin{aligned}
p_2 &= a_2 \oplus b_2 = a_2 \bar{b_2} + \bar{a}_2 b_2 = b_2 = 1 \\
g_0 &= a_0 b_0 = 0 \\
g_1 &= a_1 b_1 = b_1 = 1 \\
g_2 &= a_2 b_2 = 0 \\
g_1 &= b_3 \\
c_1 &= g_0 + c_0 p_0 = g_0 = 0 \\
c_2 &= g_1 + c_1 p_1 = g_1 = 1 \\
c_3 &= g_2 + c_2 p_2 = 0 + 11 = 1 \\
c_4 &= g_3 + c_3 p_3 = a_3 b_3 + a_3 \oplus b_3 = b_3 + \bar{b_3} = 1.
\end{aligned}
$$

So, we get the answer $C_4 = 1$.