National Chiao Tung University
Dept. of Computer & Information Science
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Handout #3
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Simple Solutions to Homework #1

1. Solution 1–7 (page 27).

Decimal	Binary	Octal	Hexadecimal
369.3125	101110001.0101	561.24	171.5
189.625	10111101.101	275.5	BD.A
214.625	11010110.101	326.5	D6.A
62407.625	1111001111000111.101	171707.5	F3C7.A

2. Solution 1–8 (b) and (c) (page 27).

	Add	Subtract	Multiply
$(15F)_{16}$ and $(A7)_{16}$	$(206)_{16}$	$(B8)_{16}$	$(E4F9)_{16}$
$(110101)_2$ and $(110110)_2$	$(1101011)_2$	$-(000001)_2$	$(101100101110)_2$

3. Solution 1–10 (page 27).
For the first method,

$$(N)_{10} \quad = \quad ((((\cdots(A_{n-1}\dot{r} + A_{n-2})\cdots)r + A_2)r + A_1)r + A_0.$$

There will be $2\lceil \log_2 N \rceil - 2$ additions and multiplications. This can be seen from the following example:

$$N \quad = \quad 6 = (110)_2.$$

N occupies 3 bits. $\lceil \log_2 6 \rceil = 3$.
$\#\ of\ Additions + \#\ of\ Multiplications = 4$.

In general, for any number with $n = 3$ the following expression can be applied:

$$((A_2 \times r) + A_1)r + A_0$$

The expression clearly indicates that two multiplications and two additions, $n-1$ and $n-1$, respectively, are needed. In the method described in the book, the number of additions remains $n - 1$; however, the number of multiplications is significantly larger. This is best shown by taking $N = 6$ ($n = 3$) as an example:

$$A_2 \times r^2 + A_1 \times r^1 + A_0 \times r^0 = A_2 \times r \times r + A_1 \times r + A_0$$

The number of multiplies is thus equal to $2 + 1 + 0$. For any n, this sum will be equal to $\frac{n(n-1)}{2}$.
$\#\ of\ Additions + \#\ of\ Multiplications = (n-1) + \frac{n(n-1)}{2} = \frac{(n-1)(n+2)}{2}$

Replacing $\lceil log_2 N \rceil$ in the equation above with n and solving the inequality

$$2(n-1) \quad \leq \quad \frac{(n-1)(n+2)}{2},$$

it can be seen that the algorithm mentioned here requires fewer operations for n greater than or equal to 3.

4. Solution 1–15 (a) and (b) (page 28).

   (a) $11 \times r + 1 \times r^0 = 144 \Longrightarrow r = 13$.

   (b) $4 \times r^2 + 3 \times r^1 + 6 \times r^0 = 357 \Longrightarrow r = 9$.

5. Solution 1–17 (page 28).
   Consider two BCD numbers, $A = A_{n-1} \cdots A_1 A_0$ and $B = B_{n-1} \cdots B_1 B_0$, where A is greater than or equal to B and A_i and B_i are BCD digits. Subtraction proceeds as follows for each digit position from 0 to n:

   (1) Subtract B_i from A_i.

   (2) If a borrow occurs, subtract 6 (0110) from the digit result in position i and subtract the borrow from A_{i+1}.

   The correction in step 2 subtracts 6 to correct the value of the borrow from value 16 in binary to value 10 in decimal.

6. Solution 1–18 (b) (page 28).

   (b) $(111010101.11)_2$

7. Solution 1–20 (page 28).

   John Doe

8. Solution 1–21 (a), (b), (c), and (d) (page 28).

Binary	BCD	ASCII			Unicode
100100111	0010 1001 0101	011 0010	011 1001	011 0101	$(0032)_{16}$ $(0039)_{16}$ $(0035)_{16}$

9. Solution 1–22 (a), (b), and (c) (page 28).

   (a) $2^{32} = 4,294,967,296$ integers.

   (b) $32\ bits \times (1\ digit)/(4\ bits) = 8\ digits = 10^8 = 100,000,000$ integers.

   (c) $32\ bits \times (1\ digit)/(8\ bits) = 4\ digits = 10^4 = 1,000$ integers.

10. Solution 1–26 (pages 28–29).

    $1 \div 4 = \frac{1}{4}$ is a FRACTION.

11. Solution 2–2 (c) and (d) (page 92).

    (c)

$$\begin{aligned} \bar{A} + AB + A\bar{C} + A\bar{B}\bar{C} &= \bar{A} + AB + A(\bar{C} + \bar{B}\bar{C}) \\ &= \bar{A} + AB + A(\bar{C}(1 + \bar{B})) \\ &= \bar{A} + A(B + \bar{C}) \\ &= (\bar{A} + A)(\bar{A} + B + \bar{C}) \\ &= \bar{A} + B + \bar{C} \end{aligned}$$

    (d)

$$\begin{aligned} Y\bar{Z} + \bar{X}\bar{Z} + \bar{X}\bar{Y} &= Y\bar{Z} + \bar{X}\bar{Y} + \bar{X}\bar{Z}(Y + \bar{Y}) \\ &= Y\bar{Z} + \bar{X}\bar{Y} + \bar{X}\bar{Z}Y + \bar{X}\bar{Z}\bar{Y} \\ &= Y\bar{Z}(1 + \bar{X}) + \bar{X}\bar{Y}(1 + \bar{Z}) \\ &= Y\bar{Z} + \bar{X}\bar{Y} \end{aligned}$$

12. Solution 2–3 (a) and (c) (page 92).

(a)

$$\begin{aligned}
& A\bar{B} + \bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}D + \bar{A}\bar{B}C\bar{D} \\
= \; & A\bar{B} + \bar{A}BC\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}D \\
= \; & A\bar{B} + (\bar{A}BC\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D}) + (\bar{A}\ \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D) + (\bar{A}\bar{B}C\bar{D} + \bar{A}\bar{B}CD) \\
= \; & A\bar{B} + \bar{A}\bar{C}\bar{D} + (\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C) \\
= \; & A\bar{B} + \bar{A}\bar{B} + \bar{A}\bar{C}\bar{D} \\
= \; & \bar{B} + \bar{A}\bar{C}\bar{D}
\end{aligned}$$

(c)

$$\begin{aligned}
& CD + A\bar{B} + AC + \bar{A}\bar{C} + \bar{A}B + \bar{C}\bar{D} \\
= \; & \overline{\overline{CD + A\bar{B} + AC + \bar{A}\bar{C} + \bar{A}B + \bar{C}\bar{D}}} \\
= \; & \overline{(\bar{C} + \bar{D})(\bar{A} + B)(\bar{A} + \bar{C})(A + C)(A + \bar{B})(C + D)} \\
= \; & \overline{(\bar{C}D + C\bar{D})(\bar{A}C + A\bar{C})(\bar{A}\bar{B} + AB)} \\
= \; & \overline{(\bar{A}C\bar{C}D + \bar{A}CCD + A\bar{C}\bar{C}D + A\bar{C}CD)(\bar{A}\bar{B} + AB)} \\
= \; & \overline{(A\bar{C}D + \bar{A}C\bar{D})(\bar{A}\bar{B} + AB)} \\
= \; & \overline{A\bar{C}D(\bar{A}\bar{B} + AB) + \bar{A}C\bar{D}(\bar{A}\bar{B} + AB)} \\
= \; & \overline{ABC\bar{D} + \bar{A}\bar{B}C\bar{D}} \\
= \; & (\bar{A} + \bar{B} + C + \bar{D})(A + B + \bar{C} + D)
\end{aligned}$$

13. Solution 2–6 (a) and (c) (page 93).

(a)

$$\begin{aligned}
\bar{X}\bar{Y} + XYZ + \bar{X}Y &= \bar{X} + XYZ \\
&= (\bar{X} + XY)(\bar{X} + Z) \\
&= (\bar{X} + X)(\bar{X} + Y)(\bar{X} + Z) \\
&= (\bar{X} + Y)(\bar{X} + Z) \\
&= \bar{X} + YZ
\end{aligned}$$

(c)

$$\begin{aligned}
\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ) &= \bar{W}X\bar{Z} + \bar{W}X\bar{Y}Z + WX + \bar{W}XYZ \\
&= WX + \bar{W}X\bar{Z} + \bar{W}XZ \\
&= WX + \bar{W}X = X
\end{aligned}$$

14. Solution 2–7 (a) and (b) (page 93).

(a) $F = \bar{A}\bar{B} + AB + \bar{B}C = \overline{\overline{\bar{A}\bar{B}}} + \overline{\overline{AB}} + \overline{\overline{\bar{B}C}} = \overline{(A + B)} + \overline{(\bar{A} + \bar{B})} + \overline{(B + \bar{C})}$

(b) $F = \overline{\overline{\bar{A}\bar{B} + AB + \bar{B}C}} = \overline{\overline{\bar{A}\bar{B}} \cdot \overline{AB} \cdot \overline{\bar{B}C}}$

15. Solution 2–8 (c) and (d) (page 93).

(c) $\bar{F} = [\bar{W} + \bar{X} + (Y + \bar{Z})(\bar{Y} + Z)][W + X + Y\bar{Z} + \bar{Y}Z]$

(d) $\bar{F} = \bar{A}B\bar{C} + (A + B)\bar{C} + \bar{A}(B + C)$