Comments

Comment on "Generic Universal Switch Blocks"

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Abstract—In the paper [5], the authors defined the well-structured symmetric switch block $M_{N,W}$ and showed that $M_{N,W}$ is universal for any pair of positive integers N and W. However, we find that this result is partially correct. Here, we show that, when $N \geq 7$, $M_{N,W}$ is not universal for odd Ws (≥ 3) and it is universal for any even W.

Index Terms—Field programmable gate array, universal switch block design, FPGA routing.

1 Nonuniversal $M_{N,W}$s

This paper concerns the design of generic switch blocks, which can be used in the two or higher dimensional FPGA architectures.

An N-sided switch block with W terminals on each side (denoted by (N, W)-SB) is said to be universal if every set of (2-pin) nets satisfying the dimension constraint (i.e., the number of nets routed through each side cannot exceed W) is simultaneously routable through the switch block. Experiments show that using universal switch blocks (USB) in an FPGA architecture results in higher routing capacity. Therefore, it is desirable, in general, to design an (N, W)-USB for each pair of positive integers $N \geq 2$ and $W \geq 1$. This problem was first proposed and solved for $N = 4$ in [3], then extended to $k \leq 6$ in [2], and, finally, is claimed to be solved in [5] by showing that the proposed symmetric switch block $M_{N,W}$ is universal for any pair of N and W.

However, we find that $M_{N,W}$ is not universal when $N \geq 7$ and W is odd (≥ 3). We will show this by presenting unroutable routing requirement (counter examples) for such cases. For example, Fig. 1a shows such a routing requirement for (7, 3)-SB, with routing requirement vector (RRV) V_0: $n_{12} = 1, n_{23} = 1, n_{24} = 1, n_{34} = 2, n_{15} = 1, n_{56} = 1, n_{57} = 1, n_{67} = 2$ and others $n_{ij} = 0$. V_0 is not routable in $M_{7,3}$ because $M_{7,3}$ is isomorphic to the disjoint union of $M_{7,2}$ and $M_{7,1}$ (see Fig. 2), but V_0 cannot be decomposed into two RRVs that are routable in $M_{7,2}$ and $M_{7,1}$. Moreover, we find that $M_{N,W}$ is universal if and only if $N \leq 6$ or W is even.

In order to give a simple proof and to employ some known graph theory results, we use graph models to represent routing requirements and switch blocks.

We label the sides of an (N, W)-SB by $1, 2, \ldots, N$ and let $t_{i,j}$ denote the jth terminal on side i, $i = 1, \ldots, N, j = 1, \ldots, W$. With these notations, a 2-pin net through the SB can be represented by a 2-sized subset of $\{1, 2, \ldots, N\}$. For example, a net spanning sides 1 and 2 corresponds to $\{1, 2\}$. A routing requirement for the SB can be represented by a collection (multiset) of 2-sized subsets of $\{1, \ldots, N\}$, which is called an N-way *global routing* with density d ((N, d)-GR for short), where d is the maximum number of occurrence of an element of $\{1, \ldots, N\}$ in the collection. Clearly, an RRV can be transformed to an N-way global routing by changing each component n_{ij} in the RRV to n_{ij} copies of $\{i, j\}$ and vice versa. An (N, d)-GR can be viewed as a multiple graph by taking its 2-sized subsets as edges. Fig. 1b shows the graph representation of the routing requirement given in Fig. 1a. An (N, W)-SB can also be viewed as a graph with $t_{i,j}$s as vertices and switches as edges. Then, a detailed routing of a net in the SB corresponds to an edge in the graph of (N, W)-SB. A detailed routing of a global routing corresponds to a set of independent edges. Under these models, the switch block design problem becomes a graph design problem.

For the sake of regularity, we add some singletons (sets of size 1) to an (N, d)-GR such that the number of sets containing each $i \in \{1, \ldots, N\}$ is equal to d. We refer to such a collection as a *balanced global routing* ((N, d)-BGR).

An (N, d)-BGR is said to be a *minimal* BGR (MBGR) if it does not contain a subglobal routing (N, d')-BGR with $d' < d$. An (N, d)-BGR is said to be a *primitive BGR* (PBGR) if it does not contain two unequal singletons. If a BGR, say R, is not primitive, then we can connect two unequal singletons in R and obtain a BGR with a smaller number of unequal singletons. Continuing this process, we will finally derive a PBGR, say R'. Any detailed routing of R' induces a detailed routing of R by simply deleting the edges representing the 2-sized sets in R' which were obtained by combining the unequal singletons in R. An (N, d)-PBGR with $d \leq W$ can also be converted into an (N, W)-PBGR by adding singletons and connecting unequal singletons. Therefore, in the designing of a universal (N, W)-SB, we can only consider the routability for all (N, W)-PBGRs.

The BGR representation has two advantages. First, an (N, d)-BGR GR corresponds to a regular hypergraph with vertex set $\{1, \ldots, N\}$ and edge set GR. Here, by regular we mean the degrees of all vertices are equal; the degree of a vertex is defined to be the number of edges incident with it. We refer to such a hypergraph as a 2-graph. Note that 2-graphs allow singletons. Second, the regularity of BGR leads to a precise decomposition theorem [4], which says that, for any given N, there is a finite number of N-way MBGRs and an (N, d)-BGR can be decomposed into a collection of N-way MBGRs.

The symmetric switch blocks $M_{N,W}$ are defined by the following algorithm in [5]:

Algorithm: Symmetric-Switch-Block(N, W)
Input: N—number of sides of the polygonal switch block;
W—number of terminals on each side of the switch block.
Output: $M_{N,W}(T, S)$—the N-sided symmetric switch block of size W; T: set of terminals; S: set of switches.

$T \leftarrow t_{i,j}, i = 1, 2, \ldots, N, j = 1, 2, \ldots, W$;
$S \leftarrow \emptyset$;
for $k = 1$ to $\lfloor \frac{W}{2} \rfloor$ do
  for $i = 1$ to N do
    for $j = 1$ to N do
      if $i \neq j$
        $S \leftarrow S \cup \{(t_{i,k}, t_{j,W-k+1})\}$;
if W is odd
  for $i = 1$ to N do
    for $j = 1$ to N do
      if $i \neq j$

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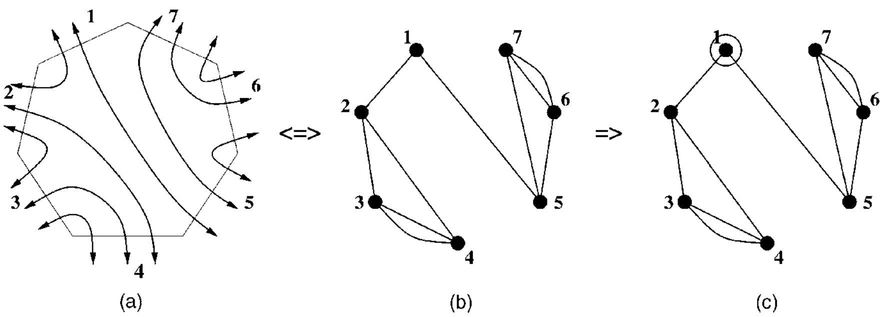

Fig. 1. Example of RRV, (7, 3)-GR and (7, 3)-PBGR. (a) Diagram of V_0. (b) Graph representation of global routing of V_0. (c) Corresponding (7, 3)-PBGR GR_0.

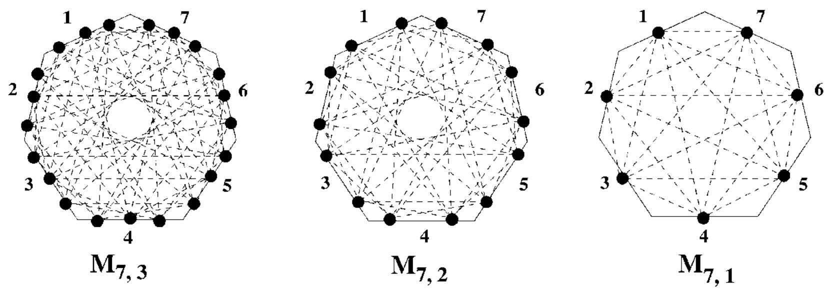

Fig. 2. $M_{7,3}$ and its decomposition.

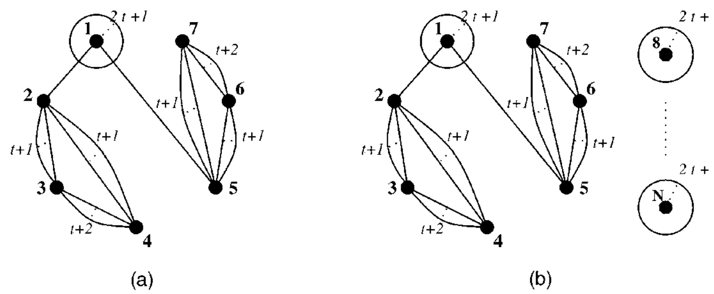

Fig. 3. (7, 2t + 3)-PMBGR, (N, 2t + 3)-PMBGR for $N \geq 8$, $t = 0, 1, \ldots$. (a) (7, 2t + 3)-BGR $GR_{7,1}$. (b) (N, 2t + 3)-BGR $GR_{N,1}$.

$S \leftarrow S \cup \{(t_{i,\lceil\frac{W}{2}\rceil}, t_{j,\lceil\frac{W}{2}\rceil})\}$;
Output $M_{N,W}(T, S)$;

$M_{N,W}$ has a very nice decomposable property. When W is even, $M_{N,W}$ is isomorphic to a disjoint union of $M/2$ copies of $M_{N,2}$; when W is odd, $M_{N,W}$ is isomorphic to a disjoint union of $\frac{M-1}{2}$ copies of $M_{N,2}$ and an $M_{N,1}$ (Lemma 1 of [5]). Fig. 2 shows $M_{7,3}$ and its decomposition.

Next, we show that $M_{N,W}$ is not universal when $N \geq 7$ and W is odd (≥ 3). Let N and W be such a pair of integers. Since $M_{N,W}$ is isomorphic to the disjoint union of $\frac{W-1}{2}$ $M_{N,2}$s and one $M_{N,1}$, it is sufficient to show the existence of (N, W)-BGRs which do not contain (N, 1)-BGRs as subglobal routings.

Fig. 1b shows the (7, 3)-GR graph corresponding to routing requirement V_0 and Fig. 1c, the 2-graph of the corresponding (7, 3)-BGR (called GR_0). Now, we show by contradiction that GR_0 does not contain a (7, 1)-BGR. Suppose GR_0 contains a (7, 1)-BGR, say GR'. Then, GR' contains exactly one of the sets $\{1\}, \{1, 2\}$, and $\{1, 5\}$. GR' cannot contain $\{1\}$ since no subset of $\{\{2, 3\}, \{2, 4\}, \{3, 4\}\}$ can cover each of 2, 3, and 4 exactly once. GR' cannot contain $\{1, 2\}$ since any subset of $\{\{5, 6\}, \{5, 7\}, \{6, 7\}\}$ cannot cover each of 5, 6, and 7 exactly once. Similarly, GR' does not contain $\{1, 5\}$. Hence, GR does not contain a (7, 1)-BGR. It follows that $M_{7,3}$ is not universal.

For $N = 7$ and $W = 2t + 3$ and $t \geq 1$, let GR_t be the (7, 2t + 3)-BGR obtained from GR_0 by adding t copies of $\{2, 3\}, \{2, 4\}, \{3, 4\}, \{5, 6\}, \{5, 7\}$, and $\{6, 7\}$ and $2t$ copies of $\{1\}$, see Fig. 3a. It can be shown similarly that GR_t does not contain a (7, 1)-PBGR. Therefore, $M_{7,2t+3}$ is not universal when $t \geq 1$.

For $N \geq 8$ and $W = 2t + 3$ and $t \geq 0$, let $GR_{N,t}$ be the (N, 2t + 3)-BGR obtained by adding N copies of singletons of $\{8\}, \ldots, \{N\}$ to GR_t, see Fig. 3b. Then, $GR_{N,t}$ does not contain an (N, 1)-BGR since, otherwise, GR_t would do. Therefore, $M_{N,2t+3}$ are not universal for all $N \geq 8$ and $t = 0, 1, \ldots$.

Summing up above, we know $M_{N,W}$ is not universal when $N \geq 7$ and W is odd (≥ 3).

2 UNIVERSAL $M_{N,W}$S

Now, the question is when is $M_{N,W}$ universal?

It was shown in [3], [2] that $M_{N,W}$ is universal when $N \leq 6$. It is also true when $W = 1, 2$ by Lemma 12 of [5]. We have just shown that $M_{N,W}$ is not universal when $N \geq 7$ and W (≥ 3) is odd. What then are the cases when $N \geq 7$ and W is even (≥ 4)? Are they universal? Fortunately, the answer to this question is yes.

We will show that the statement of Lemma 9 in [5] is true when W is even. That is, an (N, W)-PBGR can be decomposed into $\frac{W}{2}$ (N, 2)-PBGRs when W is even. The proof of Lemma 9 in [5] is flawed. Next, we give a short proof using Tutte's famous *f*-factor theorem (Corollary 3.11, p. 78 in [1]).

To describe the theorem, we need some definitions and notations. Let $G = (V, E)$ be a graph and k be a positive integer. A *k*-factor of G is a subgraph of G containing every vertex of G and with every vertex having the degree of k. Let D and S be disjoint

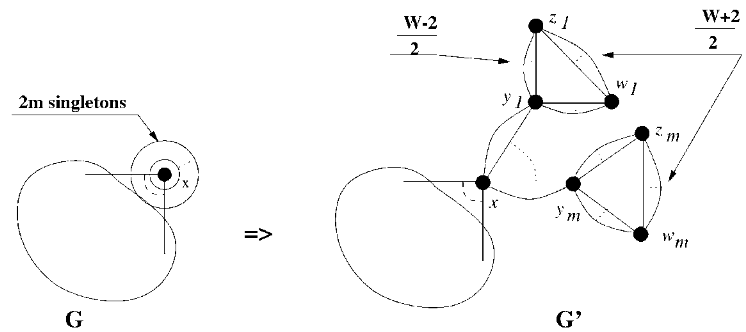

Fig. 4. Transformation of 2-graph PBGR to a multigraph.

subsets of V. $G - D$ denotes the graph obtained from G by deleting all vertices in D and $d_{G-D}(x)$ denotes the degree of vertex x in graph $G - D$. $e_G(S, D)$ denotes the number of edges of G having one end in S and the other in D.

Lemma 1 [1] (*k*-Factor Theorem). *A loop-free multigraph G contains a k-factor if and only if*

$$\Delta = k|D| - q(D, S) - \sum_{x \in S}(k - d_{G-D}(x)) \geq 0 \quad (1)$$

for all disjoint sets $D, S \subset V(G)$, *where* $q(D, S)$ *denotes the number components C of* $G - D - S$ *such that* $e_G(S, V(C)) + k|V(C)|$ *is odd.*

Corollary 1. *A regular multigraph of even degree contains a 2-factor.*

Proof. Let the degree of G be $2r$ ($r \geq 1$). Then, for any two disjoint sets $D, S \subset V(G)$, we have

$$\begin{aligned}\Delta &= 2|D| - q(D, S) - \sum_{x \in S}(2 - d_{G-D}(x)) \\ &= 2|D| - q(D, S) - 2|S| + \sum_{x \in S} d_{G-D}(x) \\ &= 2|D| - q(D, S) - 2|S| + 2r|S| - e_G(D, S) \\ &= 2|D| - 2|S| + 2r|S| - e_G(D, S) - q(D, S).\end{aligned}$$

Moreover, for a component C of $G - D - S$ with odd value of $e_G(V(C), S) + 2|V(C)|$ or, equivalently, $e_G(V(C), S)$, we have $e_G(V(C), D) \geq 1$ and $e_G(V(C), S) \geq 1$ since

$$d_G(x) = 2r, x \in V(C).$$

Then,

$$q(D, S) + e_G(D, S) \leq 2r|S|, \quad (2)$$

$$q(D, S) + e_G(D, S) \leq 2r|D|. \quad (3)$$

If $|D| \geq |S|$, by (2) we have

$$\Delta = 2(|D| - |S|) + (2r|S| - e_G(D, S) - q(D, S)) \geq 0.$$

Otherwise $|D| < |S|$, by (3) we have

$$\begin{aligned}\Delta &= 2|D| - 2|S| + 2r|S| - e_G(D, S) - q(D, S) \\ &\geq 2|D| - 2|S| + 2r|S| - 2r|D| \\ &= 2(|S| - |D|)(r - 1) \\ &\geq 0.\end{aligned}$$

Inequality (1) holds in both cases, therefore, G contains a 2-factor by Lemma 1. □

Corollary 2. *When W is even, an (N, W)-PBGR can be decomposed into* $\frac{W}{2}$ *(N, 2)-PBGRs.*

Proof. Let G be a 2-graph representation of an (*N*, *W*)-PBGR with even W. If G does not have singletons, then G is a regular multigraph of even degree. Therefore, G has a 2-factor by Corollary 1. Otherwise, G will have singletons with all of them being equal singletons, say $\{x\}$, and the number of them is an even number, say $2m$. Let G' be the regular multigraph obtained by adding 2 copies of $\{x, y_i\}$, $\frac{W-2}{2}$ copies of $\{y_i, z_i\}$ and $\{y_i, w_i\}$, and $\frac{W+2}{2}$ copies of $\{z_i, w_i\}$ for $i = 1, \ldots, m$, where y_i, z_i, w_i are new vertices (see Fig. 4). Clearly, G' has degree W and a 2-factor of G' can be boiled down to a 2-factor of G. G' contains a 2-factor by the above argument; therefore, G contains a 2-factor. Since removing the edges of a 2-factor from G results in a regular graph of even degree, it contains a 2-factor too. Continuing this process, we know G can be decomposed into union of 2-factors and, hence, an (*N*, *W*)-PBGR can be decomposed into $\frac{W}{2}$ (*N*, 2)-PBGRs. □

Now, we show $M_{N,W}$ is universal whenever W is even (≥ 4). Let W be an even number. Then, $M_{N,W}$ is isomorphic to the disjoint union of $\frac{W}{2}$ $M_{N,2}$s. By Corollary 2, every (*N*, *W*)-PBGR can be decomposed into $\frac{W}{2}$ (*N*, 2)-PBGRs, where each can be routed in an $M_{N,2}$ since every $M_{N,2}$ is universal (Lemma 12 of [5]). It follows that $M_{N,W}$ is universal.

It is known that the number of switches in $M_{N,W}$ is $\binom{N}{2}W$ and it is a lower bound for the universal (*N*, *W*)-SB. Therefore, an $M_{N,W}$ is an optimum USB if it is universal.

Summarizing the above, we know that the statement about $M_{N,W}$ in [5] should be modified to the following theorem.

Theorem 1. $M_{N,W}$ *is universal if and only if* $N \leq 6$ *or* W *is even.* $M_{N,W}$ *is an optimum universal switch block if it is universal.*

3 Conclusion

In view of practice (practical application), it is quite useful already to have the result that $M_{N,W}$ is universal for an even W since we can choose to design FPGA switch boxes with an even number of tracks to gain universal routing property. However, as a problem, the generic (*N*, *W*)-USB design problem for $N \geq 7$ and odd W (≥ 3) is still left open and it seems to be a hard problem because no efficient method is known to compute all N-way MBGRs for any given N.

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