Project 1

Using REDLIB to solve the cannibals & missioneries game.

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Project 1: Cannibals & Missioneries (1/6)

3 cannibals and 3 missionaries at a bank.
They want to cross the river.
- Only 1 boat.
- At most 2 persons in the boat at at time.
- At a bank, if there are more cannibals than missionaries, the cannibals will eat the missionaries.
- Please help the cannibals and the missionaries cross the river safely.
LEMMA: If there is a solution sequence, then the sequence is no longer than $4 \times 4 = 16$.
Proof: At any step, there can be 0, 1, 2, 3 missioneries and 0, 1, 2, 3 cannibals at the starting bank.

Representing a solution sequence with logics.
atomic propositions of the form $s(i,j,k)$
• $i \in \{0,1,2,3\}$
• $j \in \{0,1,2,3\}$
• $k \in \{1,2,\ldots,16\}$
$s(i,j,k)$ : at step $k$, there are $i$ missioneries and $j$ cannibals at the starting bank.
Project 1: Cannibals & Missioneries (4/6)
What is a safe step?
s(i,j,k) → i ≥ j

What is a safe step?

s(0,0,k) ∨ s(1,0,k) ∨ s(1,1,k)
∨ s(2,0,k) ∨ s(2,1,k) ∨ s(2,2,k)
∨ s(3,0,k) ∨ s(3,1,k) ∨ s(3,2,k) ∨ s(3,3,k)

Project 1: Cannibals & Missioneries (5/6)
What is the relation between two steps?

s(i,j,k) →
\[
\begin{align*}
& s(i-1,j,k+1) ∨ s(i,j-1,k+1) \\
& ∨ s(i-1,j-1,k+1) ∨ s(i+1,j+1,k+1) \\
& ∨ s(i-2,j,k+1) ∨ s(i,j-2,k+1) \\
& ∨ s(i+2,j,k+1) ∨ s(i,j+2,k+1)
\end{align*}
\]
Project 1: 
Cannibals & Missioneries (6/6)

• What is the initial step? \( s(3,3,1) \)
• What is the mutual exclusion of different steps?
  \( \forall i, \forall j, \forall k, (s(i,j,k) \Rightarrow \forall i', \forall j', (s(i',j',k) \Rightarrow (i = i' \land j = j')) \)

• What is a solution?
  \( \exists k, s(0,0,k), \) or we can write
  \( s(0,0,1) \lor \ldots \lor s(0,0,16) \)

Project 1 assignment

Please use REDLIB at
http://sourceforge.net/projects/redlib
to construct a program that does the following.
• The program accepts an input natural number \( n \).
• The program answer if there is a step sequence
to move \( n \) missioneries and \( n \) cannibals crossing
the river safely.
• If there is, the program also out the step
sequence.