1. Suppose we have the following theorem. (10/10)

\[ \forall x(P(x) \Rightarrow \exists y(Q(y))), \forall x(Q(x) \Rightarrow \exists y(R(y))), \forall x(R(x) \Rightarrow \exists y(S(y))) \]

\[ \models (\exists z(P(z))) \Rightarrow (\exists z(S(z))) \]

Please do the following steps.

(a) Convert the theorem for proof by refutation.

\[ \forall x (P(x) \Rightarrow \exists y(Q(y))) \land \forall x (Q(x) \Rightarrow \exists y(R(y))) \land \forall x (R(x) \Rightarrow \exists y(S(y))) \land \neg ((\exists z (P(z))) \Rightarrow (\exists z (S(z)))) \]

(b) Eliminate implication operators

\[ \forall x (\neg P(x) \lor \exists y(Q(y))) \land \forall x (\neg Q(x) \lor \exists y(R(y))) \land \forall x (\neg R(x) \lor \exists y(S(y))) \land \neg ((\exists z (P(z))) \lor (\exists z (S(z)))) \]

(c) Push all negation signs to the atom level

\[ \forall x (\neg P(x) \lor \exists y(Q(y))) \land \forall x (\neg Q(x) \lor \exists y(R(y))) \land \forall x (\neg R(x) \lor \exists y(S(y))) \land (\exists z (P(z))) \land (\forall z (\neg S(z))) \]

(d) Standardize variables

\[ \forall x1 (\neg P(x1) \lor \exists y(Q(y1))) \land \forall x2 (\neg Q(x2) \lor \exists y2(R(y2))) \land \forall x3 (\neg R(x3) \lor \exists y3(S(y3))) \land (\exists z4 (P(z4))) \land (\forall z5 (\neg S(z5))) \]

(e) Eliminate existential quantifiers using Skolemization

\[ \forall x1 (\neg P(x1) \lor Q(h1(x1))) \land \forall x2 (\neg Q(x2) \lor R(h2(x2))) \land \forall x3 (\neg R(x3) \lor S(h3(x3))) \land P(a) \land \forall z5 (\neg S(z5)) \]
(f) Convert it to the conjunctive normal form
\[
\forall x_1 \forall x_2 \forall x_3 \forall z_5 \ (¬P(x_1) \lor Q(h_1(x_1))) \\
\land (¬Q(x_2) \lor R(h_2(x_2))) \\
\land (¬R(x_3) \lor S(h_3(x_3))) \\
\land P(a) \land (¬S(z_5))
\]

(g) Eliminate the universal quantifiers
\[
(¬P(x_1) \lor Q(h_1(x_1))) \land (¬Q(x_2) \lor R(h_2(x_2))) \\
\land (¬R(x_3) \lor S(h_3(x_3))) \land P(a) \land (¬S(z_5))
\]

(h) Eliminate the conjunction operators
\[
(¬P(x_1) \lor Q(h_1(x_1))) \\
(¬Q(x_2) \lor R(h_2(x_2))) \\
(¬R(x_3) \lor S(h_3(x_3))) \\
P(a) \\
¬S(z_5)
\]

(i) Rename variables so that no variable occurs in more than one clause.
\[
¬P(x_1) \lor Q(h_1(x_1)) \\
¬Q(x_2) \lor R(h_2(x_2)) \\
¬R(x_3) \lor S(h_3(x_3)) \\
P(a) \\
¬S(z_5)
\]

(j) Use resolution principle to prove the theorem
\[
\begin{align*}
¬R(x_3) \lor S(h_3(x_3)) & \quad ¬S(z_5) \\
¬R(x_3) \lor S(h_3(x_3)) & \quad h_3(x_3) / z_5
\end{align*}
\]
\[
\begin{align*}
¬Q(x_2) \lor R(h_2(x_2)) & \quad ¬R(x_6) \\
¬Q(x_2) \lor R(h_2(x_2)) & \quad h_2(x_2) / x_6
\end{align*}
\]
\[
\begin{align*}
¬P(x_1) \lor Q(h_1(x_1)) & \quad ¬Q(x_7) \\
¬P(x_1) \lor Q(h_1(x_1)) & \quad h_1(x_1) / x_7
\end{align*}
\]
\[
\begin{align*}
¬P(x_8) & \quad P(a) \\
¬P(x_8) & \quad a / x_8
\end{align*}
\]
\[
⊥
\]
2. Assume that we have a state $s: (a=30, b=2, c=3)$ in a Kripke structure with three integer variables $a$, $b$, and $c$. Please write down the values of the following evaluations. (5/15)

(a) $\langle a+30*b, s \rangle = ? \quad 30+30*2 = 90$

(b) $\langle a+30*b < c, s \rangle = ? \quad 90<3 = false$

(c) $\langle \neg (a+b<30*c), s \rangle = ? \quad \neg \ 30+2<30 = \neg true = false$

3. Given a statement $E$ and a statement $s$, we let $s[E]$ be the result state after executing $E$ in $s$. Please write down the states of the following execution with respect to the Kripke structure in problem 2. (10/25)

(a) $(a=30, b=2, c=3)[\text{while } (a < b) \ a = (a+b+c)/b; ]$
   $(a=30, b=2, c=3)$

(b) $(a=30, b=2, c=3)[\text{while } (a > b) \ a = (a+b+c)/b; ]$
   No such state exists since this is an infinite loop.

(c) $(a=30, b=2, c=3)[\text{while } (a>b) \ a = (a+b+c) \% (2*c); ]$
   $(a=2, b=2, c=3)$

4. Suppose we have a simple C program in the following. (10/35)

```c
main () {
    int a = 3, b = 0, sum = 0;
    for (b = 0; b < 3; b++) sum = sum + a*b + 1; .......
} ..........................................................4
```

Please draw the Kripke structure of the program.
5. We have the following two Kripke structures:

Please draw the composition of the two structures. Note that in the composition, two states can be composed only if they agree in the values of the common variables. (10/45)
6. Please write LTL formulas for the following specifications. (5/50)
(The underlined phrases are atomic propositions.)

(a) Every day, I long for the true love.
   □ long for the true love

(b) One day, you will be food for monads (單細胞生物).
   ◇ food for monad

7. Please write CTL formulas for the following specifications. (5/55)
(The underlined phrases are atomic propositions.)

(a) When I find my true love, I will marry him/her by all means.
   ∀□ (find my true love → ∀◇ marry him/her)

(b) If you drink that wine, you will have 1 million fewer brain cells by tomorrow.
   ∀□ (drink that wine → ∀◇ have 1 million fewer brain cells)

8. Please write CTL* formulas for the following specifications. (5/60)
(The underlined phrases are atomic propositions.)

(a) If you try suicide too often, you go to hell eventually.
   ∀ (□◇ try suicide) → ◇ go to hell

(b) If you marry me, I will buy you a ring and be happy forever.
   ∀ □(marry me → (□ happy) ∧ ◇ buy you a ring)
9. Please construct a tree that can tell $\forall \Box(p \Rightarrow((\exists \Box q) \land \exists \Diamond r))$
from $\forall \Box(p \Rightarrow\exists((\Box q) \land r))$. (10/70)

$q$ is always true while $r$ is never true.

10. Please construct a Büchi automata for LTL formula $(\Box(p \mathcal{U} q)) \land \Diamond \Diamond r$. (10/80)
11. Please construct the closure for LTL formula \((\Box (pUq)) \land \Diamond \Box r\).
Then please construct a structure in the tableau of the formula that shows
the formula is satisfiable.  (10/90)

closure = \{ (\Box (pUq)) \land \Diamond \Box r, \Box (pUq), \Diamond \Box (pUq), pUq,
\Diamond pUq, p, q, \Diamond \Box r, \Diamond \Diamond \Box r, \Box r, r \}. 
12. Please do labeling algorithm of CTL formula \( \forall \Box (p \Rightarrow (\exists \Box q \land \exists \Diamond r)) \) on the following automata. (The formula is already the negation of the specification.) (10/100)

\[
\forall \Box (p \Rightarrow (\exists \Box q \land \exists \Diamond r)) \equiv \neg \exists \Diamond (p \land ((\neg \exists \Box q) \lor \neg \exists \Diamond r))
\]

set of subformulas = \{\neg \exists \Diamond (p \land ((\neg \exists \Box q) \lor \neg \exists \Diamond r)), \exists \Diamond (p \land ((\neg \exists \Box q) \lor \neg \exists \Diamond r)), p, ((\neg \exists \Box q) \lor \neg \exists \Diamond r), q, \neg \exists \Diamond r, \exists \Diamond r, r\}
13. Please draw a timed automaton with the following properties. (5/105)
(a) There are three control locations, \textit{idle}, \textit{waiting}, \textit{executing}.
(b) If the system is in the \textit{idle} mode, sometimes it may want to execute and thus enter the \textit{waiting} mode to wait for execution.
(c) If it is in the \textit{waiting} mode, it must execute in 5 sec. Otherwise, it goes back to the \textit{idle} mode.
(d) In the \textit{executing} mode, it must finish in 3 sec. and move back to the \textit{idle} mode.
14. Please write down TCTL formulas for the following specifications. (The underlined phrases are atomic propositions) (4/109)

(a) If you are swimming and see a piranha (食人魚), it is possible that you will be in heaven in 30 seconds.

$$\forall \square ((\text{swimming } \land \text{ see a piranha}) \rightarrow \forall \exists \leq 30 \text{ in heaven})$$

(b) When you are in heaven, if you are not happy forever, you cannot move to the hell.

$$\forall \square ((\text{in heaven } \land \neg \forall \square \text{ happy}) \rightarrow \neg \exists \square \text{ move to the hell})$$

15. Please tell me what you think of the course. What is your opinion of the course? What is your suggestion to the teacher? (1/110)

I wish the teacher can treat me ice cream in every class.
16. Please draw the composition of the following two Büchi automata. (Black boxes are accepting states.) (10/120)