Module #0: Course Overview

A few general slides about the subject matter of this course.
10 slides, ½ lecture
What is Mathematics, really?

- It’s not just about numbers!
- Mathematics is much more than that:
  
  Mathematics is, most generally, the study of any and all absolutely certain truths about any and all perfectly well-defined concepts.

- But, the concepts can relate to numbers, symbols, visual patterns, or anything!

So, what’s this class about?

What are “discrete structures” anyway?

- “Discrete” (≠ “discreet!”) - Composed of distinct, seperable parts. (Opposite of continuous.)
  
  discrete:continuous :: digital:analog

- “Structures” - objects built up from simpler objects according to a definite pattern.

- “Discrete Mathematics” - The study of discrete, mathematical objects and structures.
Discrete Structures We’ll Study

- Propositions
- Predicates
- Sets
- (Discrete) Functions
- Orders of Growth
- Algorithms
- Integers
- Proofs
- Summations
- Permutations
- Combinations
- Relations
- Graphs
- Trees

Relationships Between Structures

- “□” : □ “Can be defined in terms of”
- Groups
- Complex numbers
- Real numbers
- Natural numbers
- Integers
- Infinite ordinals
- Operators
- Functions
- Relations
- Sets
- n-tuples
- Strings
- Sequences
- Matrices
- Programs
- Proofs
- Propositions
- Bits
- Trees
- Graphs
Some Notations We’ll Learn

\[ \neg p \quad p \land q \quad p \lor q \quad p \Rightarrow q \quad p \iff q \quad \forall x \ P(x) \]
\[ \exists x \ P(x) \quad \{ a_1, \ldots, a_n \} \quad Z, N, R \quad \therefore \quad \{ \{ x \mid P(x) \} \quad x \in S \]
\[ \emptyset \quad S \subseteq T \quad |S| \quad A \cup B \quad \overline{A} \quad \bigcap_{i=1}^{n} A_i \]
\[ f : A \rightarrow B \quad f^{-1}(x) \quad f \circ g \quad [x] \quad \sum_{a \in S} a_a \quad \prod_{i=1}^{n} a_i \]
\[ O, \Omega, \Theta \quad \min, \max \quad a \mid b \quad \gcd, \text{lcm} \quad \text{mod} \quad a = b \pmod{m} \]
\[ (a_1, \ldots, a_n) \quad [a_j] \quad A^T \quad A \? B \quad A^{[n]} \quad \binom{n}{r} \]
\[ C(n; n_1, \ldots, n_m) \quad p(E \mid F) \quad R^\ast \quad \Delta \quad [a]_k \quad \deg^* (v) \]

Why Study Discrete Math?

- The basis of all of digital information processing: Discrete manipulations of discrete structures represented in memory.
- It’s the basic language and conceptual foundation of all of computer science.
- Discrete concepts are also widely used throughout math, science, engineering, economics, biology, etc., …
- A generally useful tool for rational thought!
Uses for Discrete Math in Computer Science

- Advanced algorithms & data structures
- Programming language compilers & interpreters.
- Computer networks
- Operating systems
- Computer architecture

- Database management systems
- Cryptography
- Error correction codes
- Graphics & animation algorithms, game engines
- Just about everything!

Course Outline (as per Rosen)

1. Logic (§1.1-1.4)
2. Proof methods (§1.5)
3. Set theory (§1.6-1.7)
4. Functions (§1.8)
5. Algorithms (§2.1)
6. Orders of Growth (§2.2)
7. Complexity (§2.3)
8. Number Theory (§2.4-2.6)
9. Matrices (§2.7)
10. Proof strategy (§3.1)
11. Sequences (§3.2)
12. Summations (§3.2)
13. Inductive proofs (§3.3)
14. Recursion (§3.4-3.5)
15. Combinatorics (ch. 4)
16. Probability (ch. 5)
17. Recurrences (§6.1-6.3)
18. Relations (ch. 7)
19. Graph Theory (chs. 8+9)
Topics Not Covered

Other topics we probably won’t get to this term:
21. Boolean circuits (ch. 10)
   - You’ll learn this in a digital logic course.
22. Models of computing (ch. 11)
   - Most of these are obsolete for engineering purposes now anyway
23. Linear algebra (not in Rosen, see Math dept.)
   - Matrix algebra, & general linear algebraic systems
23. Abstract algebra (not in Rosen, see Math dept.)
   - Groups, rings, fields, etc.

Course Objectives

• Upon completion of this course, the student should be able to:
  – Check the validity of simple logical arguments.
  – Check the correctness of simple algorithms.
  – Creatively construct simple valid logical arguments.
  – Creatively construct simple correct algorithms.
  – Describe the definitions and properties of a variety of specific types discrete structures.
  – Correctly read, write and analyze various types of structures using standard notations.