Homework #3 (due December 8, 2016 in-class)

1. Exercise 15.1-5 (page 370).

2. (a) Exercise 15.2-1 (page 378). (b) Exercise 15.4-1 (page 396). (c) Exercise 15.5-2 (page 404).

3. Exercise 15.3-6 (page 390).

4. Exercise 15.4-6 (page 397).

5. Let $X = x_1 x_2 \ldots x_m$ and $Y = y_1 y_2 \ldots y_n$ be two character strings. This problem asks you to find the maximum common substring length for $X$ and $Y$. Notice that substrings are required to be contiguous in the original strings. For example, photograph and tomography have common substrings ph, to, ograph, etc. The maximum common substring length is 6.

   (a) Figure 1 gives the computation of the maximum common substring length on the two strings, ABAB and BAB, similar to the table used for computing the length of LCS in class. Only partial results are given. Please complete all the entries in the table.

   (b) Dynamic programming can be used to find the longest common substring efficiently. The idea is to find length of the longest common suffix for all substrings of both strings. Suppose $\gamma = \alpha \beta$ is the concatenation of two strings; we say that $\beta$ is a suffix. Find the optimal substructure for the longest common suffix problem (a recurrence relation for $LCSuff(X, Y, i, j)$).

   (c) Define the longest common substring length $LCSubStr(X, Y, m, n)$ in term of $LCSuff(X, Y, i, j)$.

   (d) Give a dynamic programming algorithm for solving this problem. What are the time and space complexity of your algorithm?

6. EDA Rhinos and Chung-Hsin Brothers are competing for the 2016 Taiwan Professional Baseball Championship. Both teams play a series of games until one of the teams wins $n$ games. Assume that the probability of the EDA Rhinos Team winning a game is the same for each game and equal to $p$ and the probability of losing a game is $q = 1 - p$. (Hence, there are no ties.) Let $P(i, j)$ be the probability of EDA Rhinos winning the series if EDA Rhinos needs $i$ more games to win the championship and Chung-Hsin Brothers needs $j$ more games to win the championship.

   (a) Find the optimal substructure (a recurrence relation for $P(i, j)$).

   (b) The probability of the EDA Rhinos team winning a game is only 0.4. Find the probability of EDA Rhinos winning a 3-game series (i.e., 2 wins to get the championship). Based on the recurrence found in (a). (No partial credits will be given for any answer not based on the recurrence found in (a).)
(c) Give a dynamic programming algorithm for solving the problem. What are the time and space complexity of your algorithm?


11. Given $n$ items, with $i$th item worth $v_i$ dollars and weighing $w_i$ kilograms, a thief wants to take as valuable a load as possible, but he can carry at most $W$ pounds in his knapsack. Suppose this thief is allowed to take any fraction of items. Prove that this fractional knapsack problem has (1) the greedy-choice property, and (2) the optimal substructure property.

12. Alan decides to follow Mayor Ko’s footsteps to bike for 380 kilometers from Taipei to his hometown Kaohsiung to cast his first vote for the presidential election on Saturday. He needs to take a rest at a 7-11 convenience store to eat a banana and drink water to travel 20 kilometers, and his map gives the distance between each pair of convenience stores along his linear route (i.e., no detour). He wishes to make as few stops at convenience stores as possible along the way. Give an efficient algorithm to determine at which convenience stores he should stop and prove the optimality of your algorithm.

13. A sequence of $n$ operations are performed on a data structure initially in state $D_1$. The $i$th operation transforms the data structure from State $D_i$ to state $D_{i+1}$, $1 \leq i \leq n$. Let 
   \[ c(D_i) = \begin{cases} 
   2\sqrt{i}, & \text{if } i \text{ is a perfect square (i.e., } 1^2, 2^2, 3^2, \ldots \text{)} \\
   2, & \text{otherwise} 
   \end{cases} \]
   denote the cost of performing the $i$th operation, $1 \leq i \leq n$. Use the potential function 
   \[ \Phi(D_i) = i - \left(\frac{\sqrt{i} - 1}{2}\right)^2 \]
   to show that the amortized cost for performing one operation of this sequence is constant.

14. Exercise 17.4-3 (page 471).


16. (DIY Problem) For this problem, you are asked to design a problem set related to Chapter(s) 15, 16, and/or 17 and give a sample solution to your problem set. Grading on this problem will be based upon the quality of the designed problem as well as the correctness of your sample solution.