Homework #1 (due April 6, 2017 in class)

1. For optical lithography, resolution $R$ is determined by the resolution constant $k_1$, the wavelength of the illumination light $\lambda$, and the numerical aperture $NA$ which is a function of the lens and the refractive index of the medium surrounding the lens. Here, $NA = n \times \sin \theta$, where $n$ is the refractive index of the medium between the projection lens and the wafer, and $\theta$ represents the maximum incident angle formed by the exposure light with the normal to the wafer surface. For current technologies, the leading-edge scanners in IC mass-production lines used ArF 193nm excimer lasers as their light source, $\theta$ is no more than 68°, and the oil used for immersion lithography has the refractive index 1.51 (while air 1.00).

(a) What would be the achievable minimum feature sizes for immersion lithography and dry (waterless) lithography?

(b) Suppose that the cost factors over the current technology are as follows: 1.6X for 157nm optical lithography, 1.8X for double patterning technology, 2.5X for triple patterning technology, 3.1X for quadruple patterning technology, and 1.2X for oil immersion lithography. For feature patterns of sizes 14nm and 10nm, how can you realize them with the least costs?

2. An independent set of a graph $G = (V, E)$ is a subset $V' \subseteq V$ of vertices such that each edge in $E$ is incident on at most one vertex in $V'$. The Independent-Set Problem (ISP) is to find a maximum-size independent set in $G$.

(a) Give an efficient algorithm for solving the ISP on a tree.

(b) Formally describe the decision problem of ISP. Let it be ISP$_D$.

(c) The 3-CNF-SAT problem (3SAT) is defined as follows:

– **Definition:** 3SAT = $\{ < \phi > :$ boolean formula $\phi$ in 3-conjunctive normal form is satisfiable $\}$.

Prove that ISP$_D$ is NP-complete by using the reduction from 3SAT shown below. **Notice that no partial credit will be given for any reduction not from 3SAT.** (Hint: Use the reduction shown below. What is the size of the independent set and its relation with the corresponding 3SAT?)

\[(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor \neg x_3) \land (\neg x_1 \lor x_2 \lor x_3)\]

3. Are the following methods polynomial-time algorithms? Justify your answers.

(a) Given $n$ items, with $i$-th item worth $v_i$ dollars and weighing $w_i$ kg, a porter develops an $O(nW)$ algorithm to pick as valuable a load as possible, where $W$ is the maximum weight (in kg) he can carry in one load.
(b) The O(rP)-time Fiduccia-Mattheyses bi-partitioning heuristic discussed in class, where P is the number of pin terminals and r is the number of passes for this iterative heuristic.
(c) The O(EV lg C)-time network-flow based algorithm for circuit bi-partitioning, where E is the number of edges, V is the number of vertices, and C is the maximum capacity of the constructed flow network.

4. Suppose that we are given a set of n objects, where the size \( s_i \) of the \( i \)th object satisfies \( 0 < s_i < 1 \). We wish to pack all the objects into the minimum number of unit-size bins. Each bin can hold any subset of the objects whose size does not exceed 1. The first-fit heuristic, discussed in class, is used to handle this problem. It takes each object in turn and places it into the first bin that can accommodate it. Let \( S = \sum_{i=1}^{n} s_i \).

(a) Argue that the optimal number of bins required is at least \( \lceil S \rceil \).
(b) Argue that the first-fit heuristic leaves at most one bin less than half full.
(c) Prove that the number of bins used by the first-fit heuristic is never more than \( \lceil 2S \rceil \).
(d) Prove an approximation ratio of 2 for the first-fit heuristic.
(e) Give an efficient implementation of the first-fit heuristic, and analyze its running time.

5. Consider a two-dimensional version of the bin packing problem. Given a set of rectangular bins each with size \( A \times A \) and a set of items \( t_1, t_2, \ldots, t_n \) where item \( t_i \) has size \( a_i \times a_i \), the problem is to pack the items in the bin trying to minimize the number of bins used. Assume either \( a_i = a_j \) or \( a_i = 2a_j \) or \( a_j = 2a_i \), \( \forall i, j \). Extend the First-Fit-Decreasing (FFD) algorithm discussed in Unit 2 to derive an exact (optimal) algorithm. Prove the optimality of your algorithm.

6. You are given a circuit with six nodes, \( a, b, c, d, e, \) and \( f \), and five edges with edge weights \( w(a,b) = 1, w(a,e) = 3, w(b,c) = 5, w(b,f) = 7, w(c,d) = 9, w(c,e) = 2, w(d,e) = 4, w(e,f) = 6 \). Apply the Kernighan-Lin heuristic to generate a two-way balanced partition. The initial partition is \( A = \{a, b, c\} \) and \( B = \{d, e, f\} \). Does the Kernighan-Lin heuristic generate the optimal partition for this example?

7. Apply the Fiduccia-Mattheyses heuristic to the circuit below to find a balanced bipartition. The balanced criterion is the same as that defined in class. Show your work. Let the desired balance factor be 0.3 and the sizes of cells as follows: \( s(C1) = 2, s(C2) = 4, s(C3) = 6, s(C4) = 1, s(C5) = 3 \) and \( s(C6) = 5 \). The initial partition is \( A = \{1, 2, 3\} \) and \( B = \{4, 5, 6\} \).

8. Perform (a) hyperedge coarsening and (b) modified hyperedge coarsening on the following netlist: \( n_1 = \{a, c\}, n_2 = \{b, d\}, n_3 = \{c, d, f\}, n_4 = \{d, f\}, n_5 = \{f, g\}, \) and \( n_6 = \{g, h\} \). Assume that the weights of all nets are 1. Break tie in alphabetical order. How many clusters are generated? Show your work.
9. For the configuration shown below, let nodes $a$ and $b$ be fixed nodes that are placed at the given locations, $c$ and $d$ be movable nodes that can be moved to the center of either Partition $A$ or Partition $B$, net $n_1$ connects nodes $a$ and $c$, and net $n_2$ connects nodes $b$ and $d$. The center-to-center distance for the two partitions $A$ and $B$ is 16. Please find the cut weights for nets $n_1$ and $n_2$ to establish an exact net weight model to capture the wirelength cost precisely.

10. (DIY Problem) For this problem, you are asked to design a problem set related to the course content covered so far and give a sample solution to your problem set. Grading on this problem will be based upon the quality of the designed problem as well as the correctness of your sample solution.