Chapter 9 Signal-flow graphs and applications

9.1 Definitions and manipulation of signal-flow graphs
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9.2 Signal-flow graph representation of a voltage source
9.3 Signal-flow graph representation of a passive single-port device
9.4 Power gain equations
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9.1 Definitions and manipulation of signal-flow graphs

Basics

1. Definitions:

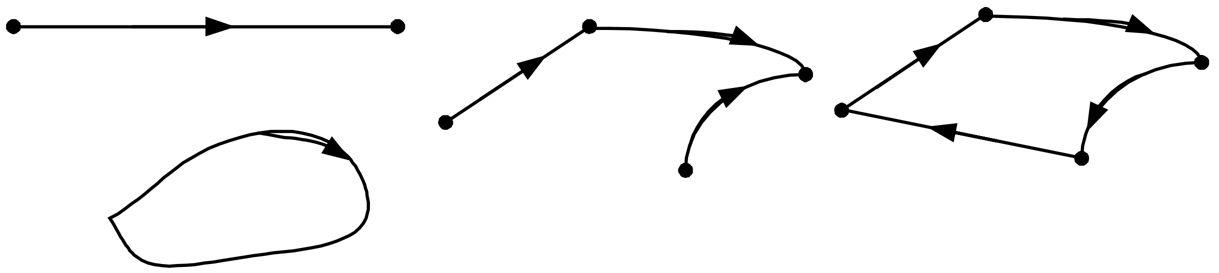
signal-flow graph: representation of a linear system

node (input and output nodes) : representation of a variable

branch: representation of direction and relation between nodes

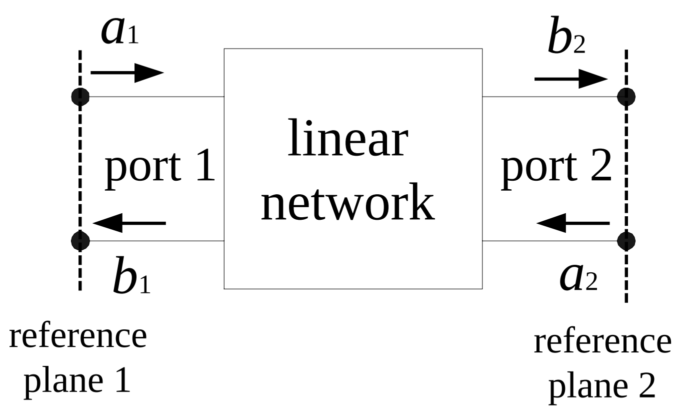
path: a continuous succession of branches traversed in the same direction

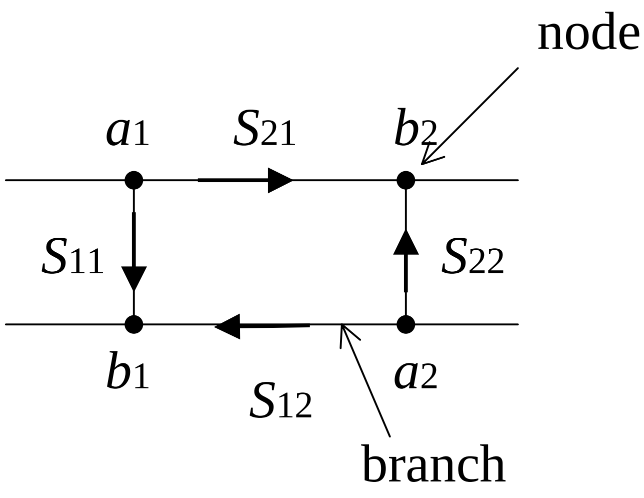
loop: a path originates and ends at the same node without encountering other nodes more than once along its traverse



<u>Discussion</u>

1. Scattering parameters

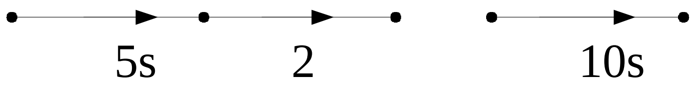




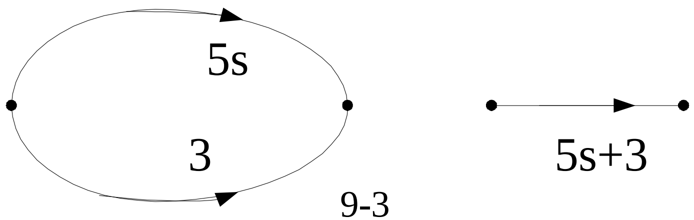
$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad \begin{aligned} b_1 &= S_{11}a_1 + S_{12}a_2 \\ b_2 &= S_{21}a_1 + S_{22}a_2 \end{aligned}$$

2. Reduction rules

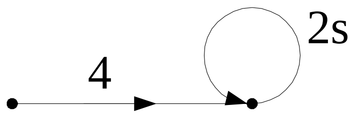
(1) Rule 1

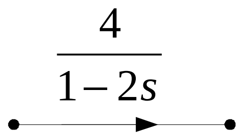


(2) Rule 2



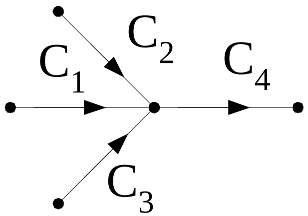
(3) Rule 3

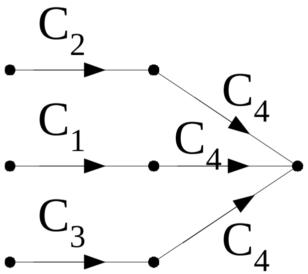




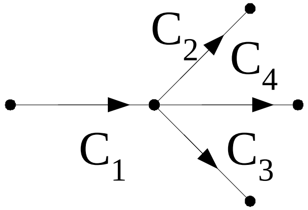
$$b = 4a + 2sb \rightarrow b - 2sb = 4a \rightarrow b = \frac{4a}{1-2s}$$

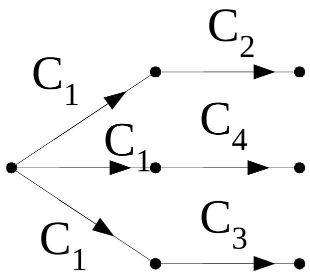
(4) Rule 4





(5) Rule 5





3. Mason's gain rule:

$$\text{transfer function } T(s) = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + ...}{\Delta}$$

P_i : gain of the ith forward path

$$\Delta = 1 - \sum L(1) + \sum L(2) - \sum L(3) + ...$$

$$\Delta_1 = 1 - \sum L(1)^{(1)} + \sum L(2)^{(1)} - \sum L(3)^{(1)} + ...$$

$$\Delta_2 = 1 - \sum L(1)^{(2)} + \sum L(2)^{(2)} - \sum L(3)^{(2)} + ...$$

$\sum L(1)$: sum of all first-order loop gains

$\sum L(2)$: sum of all second-order loop gains

$\sum L(1)^{(1)}$: sum of all first-order loop gains that do not touch path P_1 at any node

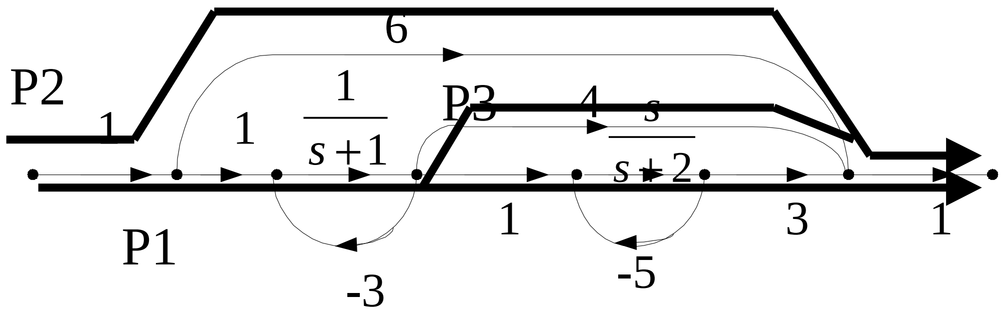
$\sum L(2)^{(1)}$: sum of all second-order loop gains that do not touch path P_1 at any node

$\sum L(1)^{(2)}$: sum of all first-order loop gains that do not touch path P_2 at any node

2nd order loop gain : product of 2 first order loops that don't touch at any point

3rd order loop gain : product of 3 first order loops that don't touch at any point

4. Ex. 9.5 find the transfer function

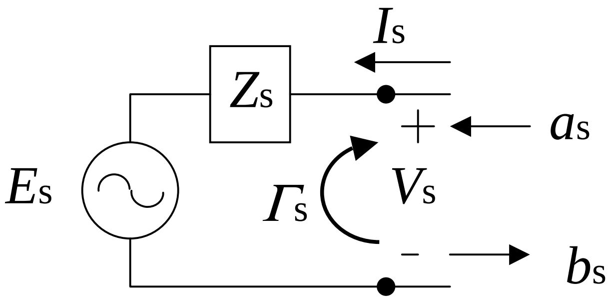


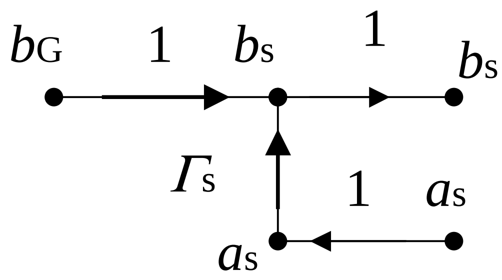
$$3\,\text{forward paths}\begin{cases} P_1 = 1\times1\times\dfrac{1}{s+1}\times1\times\dfrac{s}{s+2}\times3\times1 = \dfrac{3s}{(s+1)(s+2)} \\ P_2 = 1\times6\times1 = 6 \\ P_3 = 1\times1\times\dfrac{1}{s+1}\times(-4)\times1 = \dfrac{-4}{s+1} \end{cases}$$

$$2\,\text{loops}\begin{cases} L_1 = \dfrac{-3}{s+1} \\ L_2 = \dfrac{-5s}{s+2} \end{cases}$$

$$H(s) = \frac{P_1 + P_2(1-L_1-L_2+L_1L_2)+P_3(1-L_2)}{1-L_1-L_2+L_1L_2}$$

9.2 Signal-flow graph representation of a voltage source





$$E_s = -I_s Z_s + V_s = -(I_{s,in} + I_{s,ref})Z_s + (V_{s,in} + V_{s,ref})$$

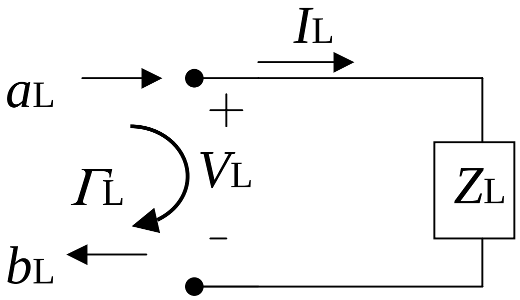
$$= -\frac{V_{s,in} - V_{s,ref}}{Z_o} Z_s + V_{s,in} + V_{s,ref} = (1 - \frac{Z_s}{Z_o})V_{s,in} + (1 + \frac{Z_s}{Z_o})V_{s,ref}$$

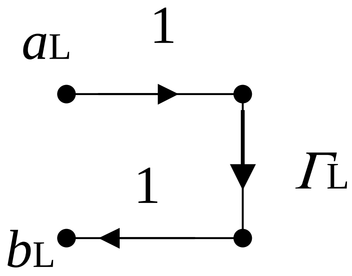
$$\rightarrow V_{s,ref} = \frac{Z_o}{Z_o + Z_s} E_s - \frac{Z_o - Z_s}{Z_o + Z_s} V_{s,in}$$

$$\rightarrow \frac{V_{s,ref}}{\sqrt{2Z_o}} = \frac{Z_o}{Z_o + Z_s} \frac{E_s}{\sqrt{2Z_o}} + \frac{Z_s - Z_o}{Z_o + Z_s} \frac{V_{s,in}}{\sqrt{2Z_o}}$$

$$\rightarrow b_s = b_G + \Gamma_s a_s$$

9.3 Signal-flow graph representation of a passive single-port device





$$V_L = I_L Z_L = (I_{L,in} + I_{L,ref}) Z_L = \frac{V_{L,in} - V_{L,ref}}{Z_o} Z_L$$

$$= V_{L,in} + V_{L,ref}$$

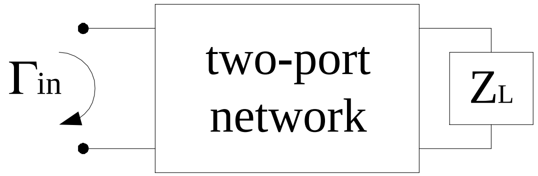
$$\rightarrow V_{L,ref} = \frac{Z_L - Z_o}{Z_L + Z_o} V_{L,in}$$

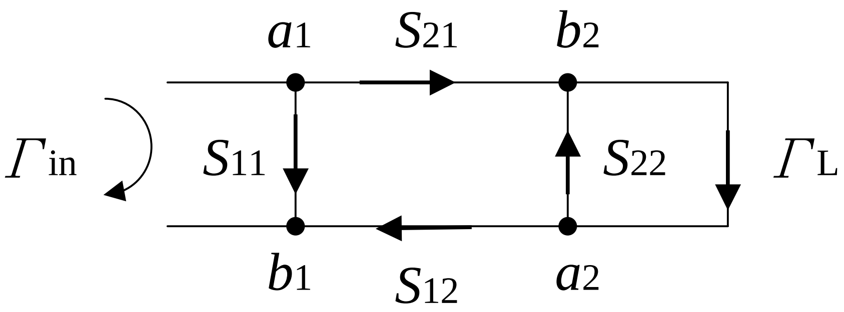
$$\rightarrow \frac{V_{L,ref}}{\sqrt{2Z_o}} = \frac{Z_L - Z_o}{Z_L + Z_o} \frac{V_{L,in}}{\sqrt{2Z_o}}$$

$$\rightarrow b_L = \Gamma_L a_L$$

Discussion

1. Ex.9.6 find Γ_{in}





$$(1)\ \Gamma_{in} = S_{11} + S_{21}\Gamma_L S_{12} + S_{21}\Gamma_L S_{12} S_{22}\Gamma_L + S_{21}\Gamma_L S_{12} S_{22}\Gamma_L S_{22}\Gamma_L + \dots.$$

$$= S_{11} + S_{21}\Gamma_L S_{12}(1 + S_{22}\Gamma_L + \dots) = S_{11} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}$$

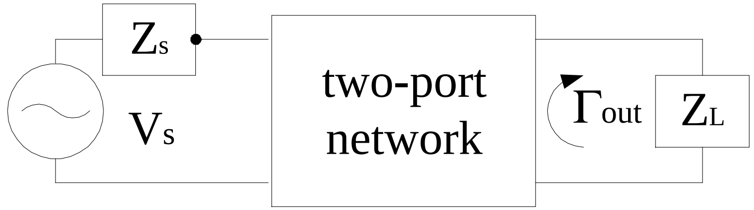
(2)Mason's rule

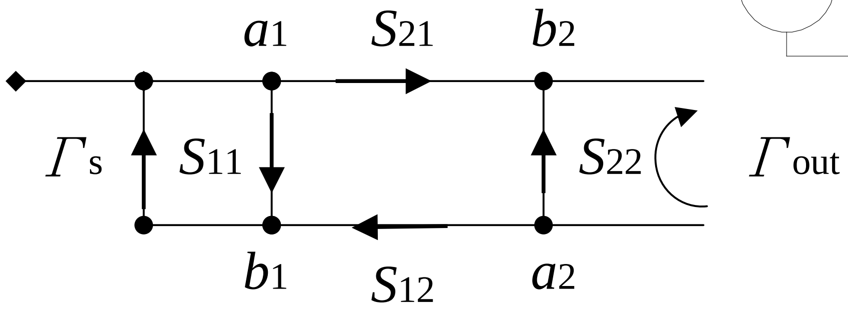
$$2 \text{ forward paths} \begin{cases} P_1 = S_{11} \\ P_2 = S_{21}\Gamma_L S_{12} \end{cases}$$

$$1 \text{ loop } L_1 = S_{22}\Gamma_L$$

$$\Gamma_{in} = \frac{P_1(1 - L_1) + P_2}{1 - L_1} = \frac{S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L} = S_{11} + \frac{S_{21}\Gamma_L S_{12}}{1 - S_{22}\Gamma_L}$$

2. Ex.9.7 find Γ_{out}





$$(1)\ \Gamma_{out} = S_{22} + S_{21}\Gamma_s S_{12} + S_{21}\Gamma_s S_{12} S_{11}\Gamma_s + S_{21}\Gamma_L S_{12} S_{11}\Gamma_s S_{11}\Gamma_s + \ldots.$$

$$= S_{22} + S_{21}\Gamma_s S_{12}(1 + S_{11}\Gamma_s + \ldots) = S_{11} + \frac{S_{21}\Gamma_s S_{12}}{1 - S_{11}\Gamma_s}$$

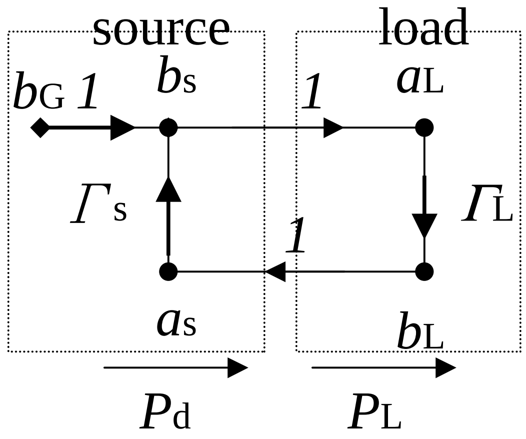
(2)Mason's rule

$$2\ \text{forward paths} \begin{cases} P_1 = S_{22} \\ P_2 = S_{21}\Gamma_s S_{12} \end{cases}$$

$$1\ \text{loop}\ L_1 = S_{11}\Gamma_s$$

$$\Gamma_{out} = \frac{P_1(1 - L_1) + P_2}{1 - L_1} = \frac{S_{22}(1 - S_{11}\Gamma_s) + S_{21}\Gamma_s S_{12}}{1 - S_{11}\Gamma_s} = S_{22} + \frac{S_{21}\Gamma_s S_{12}}{1 - S_{11}\Gamma_s}$$

3. Ex.9.8 find P_d:power delivered from source, P_L:power delivered to the load, P_{avs}:maximum power available from source



$$b_s = b_G + a_s\Gamma_s = b_G + b_L\Gamma_s = b_G + a_L\Gamma_L\Gamma_s = b_G + b_s\Gamma_L\Gamma_s$$

$$\rightarrow \begin{cases} b_s = \dfrac{b_G}{1-\Gamma_L\Gamma_s} \\ a_s = \dfrac{b_s - b_G}{\Gamma_s} = \dfrac{1}{\Gamma_s}\left(\dfrac{b_G}{1-\Gamma_L\Gamma_s} - b_G\right) = \dfrac{b_G\Gamma_L}{1-\Gamma_L\Gamma_s} \end{cases}$$

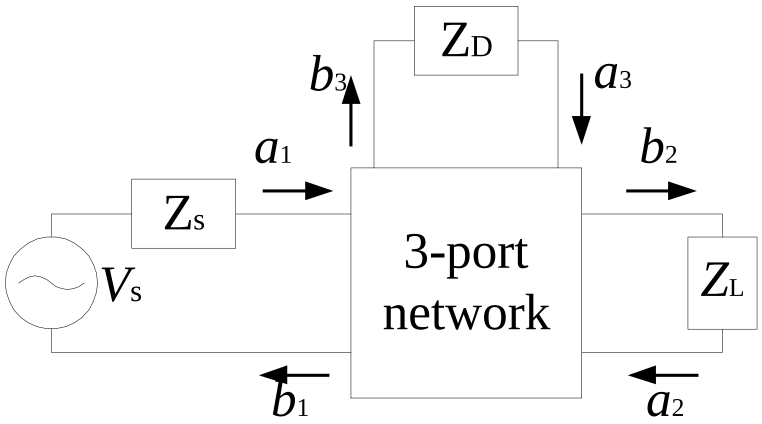
$$P_d = |b_s|^2 - |a_s|^2 = \left|\frac{b_G}{1-\Gamma_L\Gamma_s}\right|^2 - \left|\frac{b_G\Gamma_L}{1-\Gamma_L\Gamma_s}\right|^2 = \left|\frac{b_G}{1-\Gamma_L\Gamma_s}\right|^2 (1-|\Gamma_L|^2) = |b_s|^2(1-|\Gamma_L|^2) = P_L$$

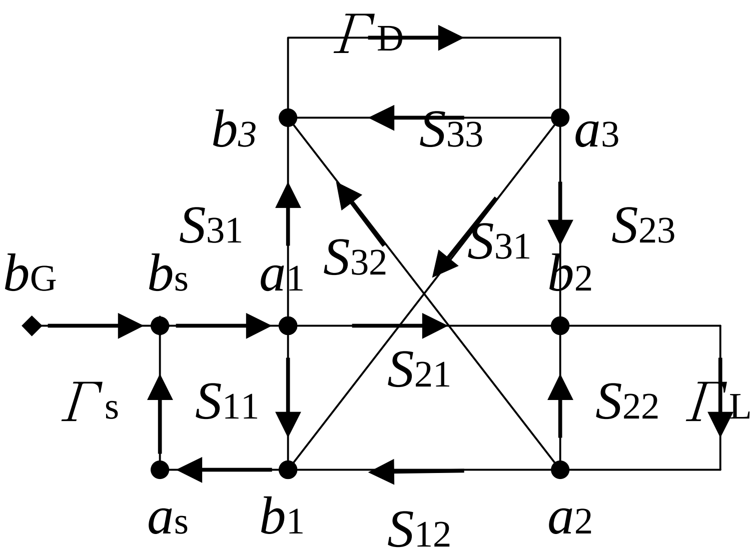
$$P_L = |a_L|^2 - |b_L|^2 = |a_L|^2(1-|\Gamma_L|^2) = |b_s|^2(1-|\Gamma_L|^2)$$

conjugate match condition : $\Gamma_s = \Gamma_L^*$

$$P_{avs} = P_d\big|_{\Gamma_s=\Gamma_L^*} = \left|\frac{b_G}{1-|\Gamma_s|^2}\right|^2 (1-|\Gamma_s|^2) = \frac{|b_G|^2}{1-|\Gamma_s|^2}$$

4. Ex.9.9 find b_3/b_s





$$\text{2 forward paths}\begin{cases} P_1 = S_{31} \\ P_2 = S_{21}\Gamma_L S_{32} \end{cases}$$

$$\text{8 loop}\begin{cases} L_1 = S_{11}\Gamma_s \\ L_2 = S_{22}\Gamma_L \\ L_3 = S_{22}\Gamma_D \\ L_4 = S_{21}S_{12}\Gamma_s\Gamma_L \\ L_5 = S_{31}S_{23}S_{12}\Gamma_s\Gamma_L\Gamma_D \\ L_6 = S_{13}S_{21}S_{32}\Gamma_s\Gamma_L\Gamma_D \\ L_7 = S_{23}S_{32}\Gamma_L\Gamma_D \\ L_8 = S_{31}S_{13}\Gamma_s\Gamma_D \end{cases}$$

$$\frac{b_3}{b_s} = \frac{P_1\Delta_1 + P_2\Delta_2}{1-\sum L(1) + \sum L(2) - \sum L(3)}$$

$$\Delta_1 = 1 - S_{22}\Gamma_L, \Delta_2 = 1$$

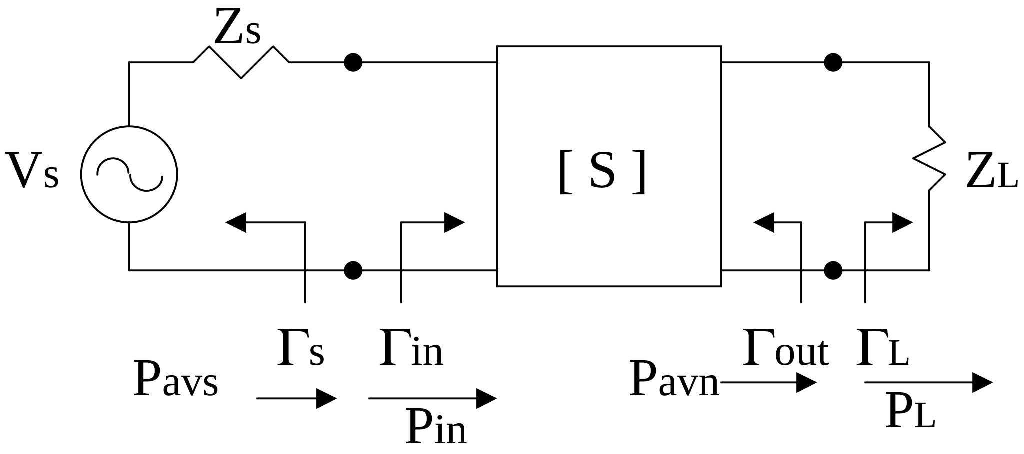
$$\sum L(1) = L_1 + \ldots + L_8$$

$$\sum L(2) = L_1L_2 + L_1L_3 + L_2L_3 + L_3L_4 + L_1L_7 + L_2L_8$$

$$\sum L(3) = L_1L_2L_3$$

9.4 Power gain equations

<u>Basics</u>

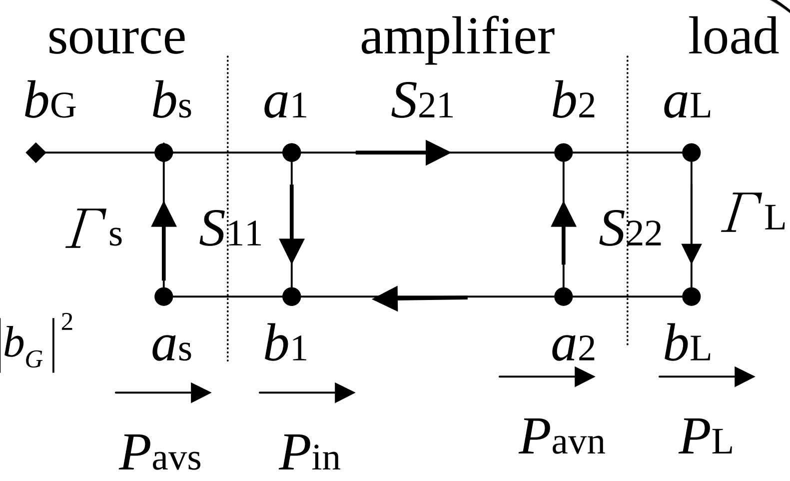


operating power gain $G_P \equiv \dfrac{P_L}{P_{in}}(S,\Gamma_L)$

available power gain $G_A \equiv \dfrac{P_{avn}}{P_{avs}}(S,\Gamma_S)$

transducer power gain $G_T \equiv \dfrac{P_L}{P_{avs}}(S,\Gamma_S,\Gamma_L)$

$P_{avs} = P_{in}\Big|_{\Gamma_{in}=\Gamma_S^*},\ P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*}$



Discussion

1.

$$\because b_s = b_G + \Gamma_s \Gamma_{in} b_s \rightarrow b_s = \frac{b_G}{1-\Gamma_s\Gamma_{in}} = a_1$$

$$P_{in} = |a_1|^2 - |b_1|^2 = |a_1|^2 (1-|\Gamma_{in}|^2) = \frac{(1-|\Gamma_{in}|^2)}{|1-\Gamma_s\Gamma_{in}|^2}|b_G|^2$$

$$\because a_L = b_2 = \frac{S_{21}a_1}{1-S_{22}\Gamma_L} = \frac{S_{21}b_G}{(1-S_{22}\Gamma_L)(1-\Gamma_s\Gamma_{in})}$$

$$P_L = |a_L|^2 - |b_L|^2 = |a_L|^2(1-|\Gamma_L|^2) = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{in}|^2}|b_G|^2$$

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|S_{21}|^2(1-|\Gamma_{out}|^2)}{|1-S_{22}\Gamma_{out}^*|^2|1-\Gamma_s\Gamma_{in}|^2}|b_G|^2, \because |1-\Gamma_s\Gamma_{in}|^2\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)^2}{|1-S_{22}\Gamma_{out}^*|^2}$$

$$= \frac{|S_{21}|^2}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}|b_G|^2$$

$$P_{avs} = P_{in}\Big|_{\Gamma_s=\Gamma_{in}^*} = \frac{1}{1-|\Gamma_s|^2}|b_G|^2$$

2.

$$P_{in} = \frac{(1-|\Gamma_{in}|^2)}{|1-\Gamma_s\Gamma_{in}|^2}|b_G|^2, P_L = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{in}|^2}|b_G|^2$$

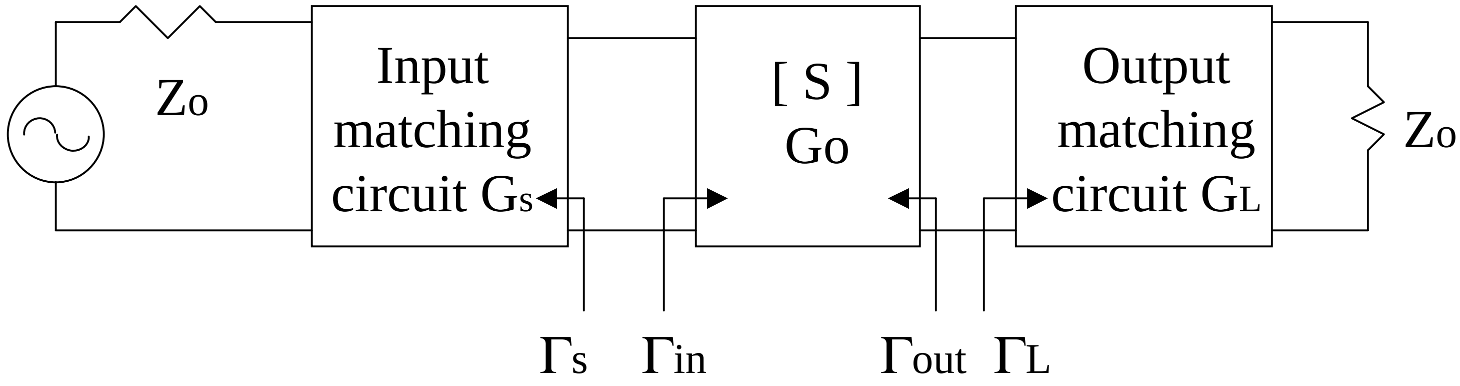
$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|S_{21}|^2}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}|b_G|^2, P_{avs} = P_{in}\Big|_{\Gamma_s=\Gamma_{in}^*} = \frac{1}{1-|\Gamma_s|^2}|b_G|^2$$

$$G_P = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2(1-|\Gamma_{in}|^2)}$$

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}$$

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-|\Gamma_s|^2)}{|1-S_{22}\Gamma_L|^2|1-\Gamma_s\Gamma_{in}|^2} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2|1-\Gamma_L\Gamma_{out}|^2}$$

5. Unilateral transducer power gain G_{TU}



$$S_{12} = 0 \rightarrow \Gamma_{\text{in}} = S_{11}$$

$$G_{TU} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = G_S G_o G_L$$

$$S_{11} = \Gamma_s^*, S_{22} = \Gamma_L^* \rightarrow G_{TU\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2 \frac{1}{1-|S_{22}|^2}$$

6. A 800MHz amplifier (Z_o=50Ω) with S_{11}=0.45∠150°, S_{12}=0.01∠-10°, S_{21}=2.0∠10°, S_{22}=0.4∠-150°, Z_s=20Ω, Z_L=30Ω → G_T, G_P, G_A

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s - Z_o} = -0.429, \Gamma_L = \frac{Z_L - Z_o}{Z_L - Z_o} = -0.25$$

$$\Gamma_{in} = S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.455\angle 150.32°, \Gamma_{out} = S_{22} + \frac{S_{21}S_{12}\Gamma_s}{1 - S_{11}\Gamma_s} = 0.408\angle 150.87°$$

$$G_P \equiv \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_L|^2} = 5.937 = 7.7dB$$

$$G_A \equiv \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)}{(1 - |\Gamma_{out}|^2)|1 - S_{11}\Gamma_s|^2} = 5.855 = 7.7dB$$

$$G_T \equiv \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_s\Gamma_{in}|^2|1 - S_{22}\Gamma_L|^2} = 5.487 = 7.4dB$$

7. A 2GHz amplifier (Z_o=50Ω) with S_{11}=0.97∠-43°, S_{12}=0,
S_{21}=3.39∠140°, S_{22}=0.63∠-32°, Γ_s=0.97∠43°, Γ_L=0.63∠32°,
→ G_T, G_P, G_A

$$S_{12} = 0 \rightarrow \Gamma_{in} = S_{11}, \Gamma_{out} = S_{22}$$

$$\Gamma_s = S_{11}^*, \Gamma_L = S_{22}^*$$

$$G_P \equiv \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_L|^2}, G_A \equiv \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)}{(1 - |\Gamma_{out}|^2)|1 - S_{11}\Gamma_s|^2}$$

$$G_T \equiv \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_s \Gamma_{in}|^2 |1 - S_{22}\Gamma_L|^2}$$

$$G_T = G_A = G_P = G_{TU\max} = \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2} = 322.42 = 25dB$$

<u>Hw #7 (due 2 weeks)</u>
5, 10, 14